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Further Developments on the (EG) Exponential-MIR Class of Distributions

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Abstract

The Modified Inverse Rayleigh (MIR) distribution appeared in [Khan, M. S. (2014).Modified inverse Rayleigh distribution. International Journal of Computer Applications, 87(13):28–33] who got some theoretical properties of this distribution, and in[Nasiru, S., Mwita, P. N. and Ngesa, O. (2017). Exponentiated Generalized Exponential Dagum Distribution. Journal of King Saud University-Science, In Press] they introduced the (EG) Exponential-X class of distributions and obtained some theoretical properties with application. By assuming the random variable X follows the MIR distribution, some theoretical properties with application of the (EG) Exponential-MIR

Class of distributions appeared in [Nasiru, S., Mwita, P. N. and Ngesa, O. (2018). Discussion on Generalized Modified Inverse Rayleigh Distribution. Applied Mathematics and Information Sciences, 12(1):113-124]. In the present paper we propose some extensions of the (EG) Exponential-MIR class of distributions. The (EG) Exponential-MIR class of distributions is part of Chapter 5 [Nasiru, S. (2018). A New Generalization of Transformed-Transformer Family of Distributions. Doctor of Philosophy thesis in Mathematics (Statistics Option). Pan African University, Institute for Basic Sciences, Technology and Innovation, Kenya], where the naming convention "NEGMIR" is used

Keywords: *T-X (W)* family of distributions; Exponentiated Generalized distributions; Modified Inverse Rayleigh distribution; biological data; health data

Introduction

T-X (W) Family of Distributions

This family of distributions is a generalization of the beta-generated family of distribu-tions first proposed by Eugene et.al [Eugene, N, Lee, C, Famoye, F: The beta-normal dis-tribution and its applications. Communications in Statistics-Theory and Methods 31(4), 497–512 (2002)]. In particular, let r(t) be the PDF of the random variable T ∈ [a, b], −∞ ≤ a < b ≤ ∞, and let *W* (*F* (*x*)) be a monotonic and absolutely continuous function of the *CDF F* (*x*) of any random variable X. The CDF of a new family of distributions defined by Alzaatreh et.al [Alzaatreh, A, Lee, C, Famoye, F: A new method for generating families of continuous distributions. Metron 71(1), 63–79 (2013b)] is given by

$$G(x)=\int_a^{W(F(x))} r(t)dt = R\{W(F(x))\}$$

Where R (•) is the CDF of the random variable T and $a \geq 0$

Remark 1.1

The PDF of the T-X (W) family of distributions is obtained by differentiating the CDF above.

Remark 1.2

When we set $W(F(x)) := -\ln(1-F(x))$ then we use the term "T-X Family of Distributions" to describe all sub-classes of the T-X (W) family of distributions induced by the weight function $W(x) = -\ln(1-x)x)$. A description of different weight functions that are appropriate given the support of the random variable T is discussed in [Alzaatreh, A, Lee, C, Famoye, F: A new method for generating families of continuous distributions. Metron 71(1), 63–79 (2013b)] A plethora of results studying properties and application of the T- X(W) family of distributions have appeared in the literature, and the research papers, assuming open access, can be easily obtained on the web via common search engines, like Google, etc.

The Exponentiated Generalized (EG) T-X family of distributions

This class of distributions appeared in [Suleman Nasiru, Peter N. Mwita and Oscar Ngesa, Exponentiated Generalized Transformed-Transformer Family of Distributions, Journal of Statistical and Econometric Methods, vol.6, no.4, 2017, 1-17] In particular the CDF Admits the following integral representation

$$G(x)=\int_0^{-\log[1-(1-\bar{F}(x)^d)^c]} r(t)dt$$

Where $c, d > 0$ and $\bar{F}(x) = 1 - F(x)$ and $F(x)$ is the CDF of a base distribution.

Remark 1.3

Note that if we set $L(x) := (1-\bar{F}(x)^d)^c$ where c, d > 0 and $\bar{F}(x) = 1 - F(x)$, and F(x) is the CDF of a base distribution, then L(x) gives the CDF of the exponentiated generalized class of distributions [G.M. Cordeiro, E.M.M. Ortega and C.C.D. da Cunha, The exponentiated generalized class of distributions, Journal of Data Science,11(1), (2013),1-27]

The (EG) Exponential-MIR family of distributions

Here we assume the random variable X follows the MIR distribution with CDF

$$F_{\alpha,\theta(x)} = e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$$

And the random variable T follows the exponential distribution with PDF

$$r_\lambda(t) = \lambda e^{-\lambda t}$$

and CDF $R_\lambda(t) = 1 - e^{-\lambda t}$

Now put $L_{(\alpha,\theta,d,c)}(x) := (1-\overline{F}_{(\alpha,\theta)}(x)^d)^c$

And observe the CDF of the (EG) Exponential-MIR family of distributions as proposed in [Nasiru, S, Mwita, P N and Ngesa, O.(2018) Discussion on Generalized Modified Inverse Rayleigh Distribution. Applied Mathematics and Information Sciences, 12(1):113-124; Nasiru, S. (2018). A New Generalization of Transformed-Transformer Family of Distributions. Doctor of Philosophy thesis in Mathematics (Statistics Option) Pan African University, Institute for Basic Sciences, Technology and Innovation, Kenya] is given by

$$G_{(\alpha,\theta,d,c,\lambda)}(x) = 1-(1-L_{(\alpha,\theta,d,c)}(x))^\lambda$$

Further Developments

In this section we present some new generalizations of the (EG) Exponential-MIR family of distributions which are induced by the other weight functions introduced in [Alzaatreh, A, Lee, C, Famoye, F: A new method for generating families of continuous distributions. Metron 71(1), 63–79 (2013b)], when the random variable T in the T-X(W) class of distributions has support $[a,\infty)$, where $a \geq 0$

Definition 2.1

A random variable Y1 will be called T – X distributed of type I if the weight function is given by

$$W(x) = -\log(1-x)$$

Definition 2.2

A random variable Y2 will be called T – X distributed of type II if the weight function is given by

$$W(x) = \frac{x}{1-x}$$

Definition 2.3

A random variable Y3 will be called T – X distributed of type III if the weight function is given by

$$W(x) = -\log(1-x^\alpha)$$

For some $\alpha > 0$

Definition 2.4

A random variable Y4 will be called T – X distributed of type IV if the weight function is given by

$$W(x) = \frac{x^\alpha}{1-x^\alpha}$$

For some $\alpha > 0$

Type II

Definition 2.5

The CDF of the (EG) Exponential-MIR class of distributions of type II has the following integral representation for $a \geq 0$

$$K_{(\alpha,\theta,d,c,\lambda)}(x) = \int_a^{\frac{L_{(\alpha,\theta,d,c)}(x)}{1-L_{(\alpha,\theta,d,c)}(x)}} r_\lambda(t)dt$$

Note that $L_{(\alpha,,\theta,1,c)}(x)$ implies the following from Definition 2.5

Corollary 2.6

The CDF of the exponentiated Exponential-MIR class of distributions of type II is given by

$$K^*_{(\alpha,\theta,c,\lambda)}(x) = R_\lambda(\frac{F^c_{(\alpha,\theta)}(x)}{1-F^c_{(\alpha,\theta)}(x)}$$

Where

$$R_\lambda(.) = 1 - e^{-\lambda(.)}$$

and

$$F_{(\alpha,\theta)}(x) = e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$$

Note that $L_{(\alpha,,\theta,1,1)}(x)$ implies the following from Definition 2.5

Corollary 2.7

The CDF of the Exponential-MIR class of distributions of type II

$$K^{**}_{(\alpha,\theta,\lambda)}(x) = R_\lambda(\frac{F_{(\alpha,\theta)}(x)}{1-F_{(\alpha,\theta)}(x)})$$

Where $R_\lambda(.) = 1 - e^{-\lambda(.)}$

and $F_{(\alpha,\theta)}(x) = e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Type III

Definition 2.8

The CDF of the (EG) Exponential-MIR class of distributions of type III has the following integral representation for $a \geq 0$ and $\xi > 0$

$$Z_{(\alpha,\theta,d,c,\lambda,\xi)}(x) = \int_a^{-\log(1-L^\xi_{(\alpha,\theta,d,c)}(x))} r_\lambda(t)dt$$

Note that $L_{(\alpha,\theta,1,c)}(x)$ implies the following from Definition 2.8

Corollary 2.9

The CDF of the exponentiated Exponential-MIR class of distributions of type III is given by

$$Z^{*}_{(\alpha,\theta,c,\lambda,\xi)}(x)=R_{\lambda}(-\log(1-F_{(\alpha,\theta)}(x)^{c\xi}(x)))$$

Where $R_{\lambda}(.)=1-e^{-\lambda(.)}$

and $F_{(\alpha,\theta)}(x)=e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Note that $L_{(\alpha,\theta,1,1)}(x)$ implies the following from Definition 2.8

Corollary 2.10

The CDF of the Exponential-MIR class of distributions of type III is given by

$$Z^{**}_{(\alpha,\theta,\lambda,\xi)}(x)=R_{\lambda}(-\log(1-F_{(\alpha,\theta)}(x)^{\xi}(x)))$$

Where $R_{\lambda}(.)=1-e^{-\lambda(.)}$

and $F_{(\alpha,\theta)}(x)=e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Type IV

Definition 2.11

The CDF of the (EG) Exponential-MIR class of distributions of type IV has the following integral representation for $\xi>0$ and $a\geq 0$

$$Q_{(\alpha,\theta,d,c,\lambda,\xi)}(x)=\int_a^{\frac{L^{\xi}_{(\alpha,\theta,d,c)}(x)}{1-L^{\xi}_{(\alpha,\theta,d,c)}(x)}} r_{\lambda}(t)dt$$

Note that $L_{(\alpha,\theta,1,c)}(x)$ implies the following from Definition 2.11

Corollary 2.12

The CDF of the exponentiated Exponential-MIR class of distributions of type IV is given by

$$Q^{*}_{(\alpha,\theta,c,\lambda,\xi)}(x)=R_{\lambda}(\frac{F_{(\alpha,\theta)}(x)^{\xi}(x)}{1-F_{(\alpha,\theta)}(x)^{\xi}(x)})$$

Where $R_{\lambda}(.)=1-e^{-\lambda(.)}$

and $F_{(\alpha,\theta)}(x)=e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Note that $L_{(\alpha,\beta,\theta,1,1)}(x)$ implies the following from Definition 2.11

Corollary 2.13

The CDF of the Exponential-MIR class of distributions of type IV is given by

$$Q^{**}_{(\alpha,\theta,\lambda,\xi)}(x)=R_{\lambda}(\frac{F_{(\alpha,\theta)}(x)^{\xi}(x)}{1-F_{(\alpha,\theta)}(x)^{\xi}(x)})$$

Where $R_{\lambda}(.)=1-e^{-\lambda(.)}$

and $F_{(\alpha,\theta)}(x)=e^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Application

In this section we compare the Exponential-Weibull class of distributions of type II and the Exponential-Weibull class of distributions of type IV in modeling the aircraft data, Table 1 [Suleman Nasiru, Peter N. Mwita and Oscar Ngesa, Discussion on Generalized Modified Inverse Rayleigh, Appl. Math. Inf. Sci. 12, No. 1, 113-124 (2018)]

Remark 3.1

When a random variable X follows the Exponential-Weibull class of distributions of type II we write $X\sim EWII(a,b,c)$

Remark 3.2

When a random variable X follows the Exponential-Weibull class of distributions of type IV we write $X\sim EWIV(a,b,c,d)$ In this section we assume the CDF of the Weibull distribution is given by

$$F_{(b,c)}(x)=1-e^{-(\frac{x}{c})^b}$$

and the CDF of the Exponential distribution is given by

$$R_{(a)}(x)=1-e^{-ax}$$

Theorem 3.3

The CDF of the Exponential-Weibull class of distributions of type II is given by

$$K^{**}_{(a,b,c)}(x)=1-e^{-ae(\frac{x}{c})^b(1-e^{-(\frac{x}{c})^b})}$$

Proof in Corollary 2.7, let $\lambda:=a$ and $F_{(\alpha,\theta)}(x):=F_{(b,c)}(x)$

Theorem 3.4

The CDF of the Exponential-Weibull class of distributions of type IV is given by

$$Q^{**}_{(a,b,c,d)}(x)=1-e^{-\frac{a(1-e^{-(\frac{x}{c})^b})^d}{1-(1-e^{-(\frac{x}{c})^b})^d}}$$

Proof In Corollary 2.13, let $\xi := d$, $\lambda := a$ and $F_{(\alpha,\theta)}(x) := F_{(b,c)}(x)$ In order to compare the two distribution models, we used the following criteria: -2(Log likelihood) and AIC (Akaike information criterion) , AICC (corrected Akaike information criterion), and BIC (Bayesian information criterion) for the data set. The better distribution corresponds to the smaller -2(Log-likelihood) AIC, AICC, and BIC values:

$$AIC = 2k - 2l$$

$$AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

$$BIC = k \log(n) - 2l$$

Where k is the number of parameters in the statistical model, n is the sample size, and l is the maximized value of the log-likelihood function under the considered model. From Table 1 above, it is clear the *EWII* (3975.82, 1.0863, 121474) distribution has the smallest AICC and BIC values, whilst the *EWIV* (216660, 1.47149, 1.21237_107, 0.681738) distribution has the smallest -2(Log-likelihood) and AIC values. When we compared the CDF's of the two distributions we obtained the following

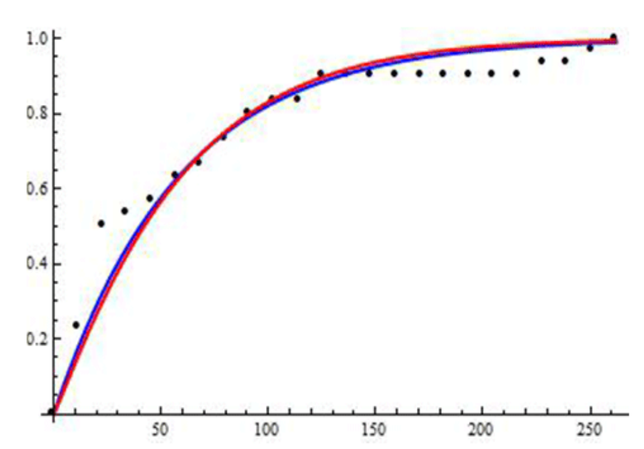

Figure 1: The CDF of EWII(3975.82, 1.0863, 121474) (red) and EWIV (216660, 1.47149, 1.21237 _ 107, 0.681738) (blue) fitted to the empirical distribution of the aircraft data, Table 3 [Suleman Nasiru, Peter N. Mwita and Oscar Ngesa, Discussion on Generalized Modified Inverse Rayleigh, Appl.Math. Inf. Sci. 12, No. 1, 113-124 (2018)]

On the other hand when we compared the PDF's of the two distributions we obtained the Following

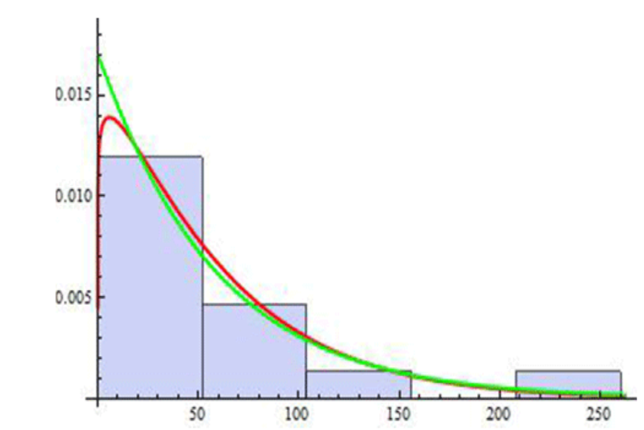

Figure 2: The PDF of *EWII* (3975.82, 1.0863, 121474) (red) and *EWIV* (216660, 1.47149, 1.21237 _ 107, 0.681738) (green) fitted to the empirical distribution of the aircraft data, Table 3 [Suleman Nasiru, Peter *N. Mwita* and *Oscar Ngesa*, Discussion on Generalized Modified Inverse Rayleigh, Appl.
Math. Inf. Sci. 12, No. 1, 113-124 (2018)]

The results from Table 1 and the Figures above, suggest the *EWIV* (216660, 1.47149, 1.21237_107, 0.681738) distribution is slightly better than the EWII (3975.82, 1.0863, 121474) distribution in modeling the aircraft data, Table 3 [Suleman Nasiru, Peter *N. Mwita* and *Oscar Ngesa*, Discussion on Generalized Modified Inverse Rayleigh, Appl. Math. Inf. Sci. 12, No. 1, 113-124 (2018)]

Concluding Remarks

Our hope is that the researchers will further develop the properties and applications of the new class of distributions presented in this paper. Finally we hope the new developments have practical significance in modeling biological data, health data, etc.

Note that $L_{(\alpha,\theta,l,c)}(x)$ implies the following from Section 1.3

Table 1: Criteria for Comparison

Model	-2(Log-likelihood)	AIC	AICC	BIC
EWII(3975.82, 1.0863, 121474)	307.351	313.351	314.274	317.554
EWIV (216660, 1.47149, 1.21237*10^7, 0.681738)	305.338	313.338	314.938	318.943

Theorem 4.1

The CDF of the exponentiated Exponential-MIR class of distributions of type I is given by

$$G^*_{(\alpha,\theta,c,\lambda)}(x) = 1-(1-F_{(\alpha,\theta)}(x)^c(x))^\lambda$$

Where $\mathbf{F}_{(\alpha,\theta)}(\mathbf{x}) = \mathbf{e}^{-(\frac{\alpha}{x}+\frac{\theta}{x^2})}$

Similarly, $L_{(\alpha,\theta,1,1)}(x)$ implies the Exponential-MIR class of distributions of type I from Section 1.3. Consequently several Corollaries can be deduced from Chapter 5[4], where they obtained several statistical/mathematical properties with application. For example, we have the following from Section 5.2 of Chapter 5[4]

Corollary 4.2

The survival function of the exponentiated Exponential-MIR class of distributions of type I is given by

$$S^*(x) = \{1-F^c_{(\alpha,\theta)}(x)\}^\lambda$$

Proof Let d = 1 in eqn (5.8) contained in Section 5.2 of Chapter 5[4]

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