

# On similarity of specific heat capacity and capillary pressure fractal dimensions for characterizing Shajara Reservoirs of the Permo-Carboniferous Shajara Formation, Saudi Arabia

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## Abstract

The quality and assessment of a reservoir can be documented in details by the application of Specific heat capacity. This research aims to calculate fractal dimension from the relationship among Specific heat capacity, maximum Specific heat capacity and wetting phase saturation and to approve it by the fractal dimension derived from the relationship among capillary pressure and wetting phase saturation. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, Specific heat capacity, maximum Specific heat capacity and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between Specific heat capacity and maximum Specific heat capacity versus logarithm wetting phase saturation. The slope of the first procedure = 3- DF (fractal dimension). The second procedure for obtaining the fractal dimension was determined by plotting the logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure = DF -3. On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units.

**Keywords:** Shajara Reservoirs; Shajara Formation; Specific Heat Capacity Fractal Dimension; Capillary Pressure Fractal Dimension

## Introduction

Seismo electric effects related to electro kinetic potential, dielectric permittivity, pressure gradient, fluid viscosity, and electric conductivity was first reported by [1]. Capillary pressure follows the scaling law at low wetting phase saturation was reported by [2]. Seismo electric phenomenon by considering electro kinetic coupling coefficient as a function of effective charge density, permeability, fluid viscosity and electric conductivity was reported by [3]. The magnitude of seismo electric current depends on porosity, pore size, zeta potential of the pore surfaces, and elastic properties of the matrix was investigated by [4]. The tangent of the ratio of converted electric field to pressure is

approximately in inverse proportion to permeability was studied by [5]. Permeability inversion from seismo electric log at low frequency was studied by [6]. They reported that, the tangent of the ratio among electric excitation intensity and pressure field is a function of porosity, fluid viscosity, frequency, tortuosity, and fluid density and Dracy permeability. A decrease of seismo electric frequencies with increasing water content was reported by [7]. An increase of seismo electric transfer function with increasing water saturation was studied by [8]. An increase of dynamic seismo electric transfer function with decreasing fluid conductivity was described by [9]. The amplitude of seismo electric signal increases with increasing permeability which means that the seismo electric effects are directly related to the permeability and can be used to study the permeability of the reservoir was illustrated by [10]. Seismo electric coupling is frequency dependent and decreases exponentially when frequency increases was demonstrated by [11]. An increase of permeability with increasing pressure head and bubble pressure fractal dimension was reported by [12, 13]. An increase of geometric relaxation time of induced polarization fractal dimension with permeability increasing and grain size was described by [14, 15].

## Materials and Methods

Sandstone samples were collected from the surface type section of the Permo-Carboniferous Shajara Formation, latitude 26° 52' 17.4", longitude 43° 36' 18". (Figure1). Porosity was measured on collected samples using mercury intrusion Porosimetry and permeability was derived from capillary pressure data. The purpose of this paper is to obtain Specific heat capacity fractal dimension and to confirm it by capillary pressure fractal dimension. The fractal dimension of the first procedure is determined from the positive slope of the plot of logarithm of the ratio of Specific heat capacity to maximum Specific heat capacity  $\log (SHC^{1/2}/SHC^{1/2max})$  versus  $\log$  wetting phase saturation ( $\log Sw$ ). Whereas the fractal dimension of the second procedure is determined from the negative slope of the plot of logarithm of capillary pressure ( $\log Pc$ ) versus logarithm of wetting phase saturation ( $\log Sw$ ).

AGE	Fm.	Mbr.	unit	LITHO-LOGY	DESCRIPTION
Late Permian	Khuff Formation	Huqayf Member			Limestone : Cream, dense, burrowed, thickness 6.56'
Late Carboniferous - Permian	Shajara Formation	Upper Shajara Member	Upper Shajara mudstone		Sub-Khuff unconformity.
					Mudstone : Yellow, thickness 17.7'
			Upper Shajar Reservoir		Sandstone : Light brown, cross-bedded, coarse-grained, poorly sorted, porous, friable, thickness 6.5'
					Sandstone : Yellow, medium-grained, very coarse-grained, poorly, moderately sorted, porous, friable, thickness 13.1'
					Mudstone : Yellow-green, thickness 11.8'
		Middle Shajara Member	Middle Shajara mudstone		Mudstone : Yellow, thickness 1.3'
					Mudstone : Brown, thickness 4.5'
			Middle Shajara Reservoir		Sandstone : Light brown, medium-grained, moderately sorted, porous, friable, thickness 3.6'
					Sandstone : Yellow, medium-grained, moderately well sorted, porous, friable, thickness 0.9'
					Sandstone : Red, coarse-grained, medium-grained, moderately well sorted, porous, friable, thickness 13.4'
Lower Shajara Member	Lower Shajara Reservoir		Sandstone : White with yellow spots, fine-grained, hard, thickness 2.6'		
			Sandstone : Limonite, thickness 1.3'		
			Sandstone : White, coarse-grained, very poorly sorted, thickness 4.5'		
			Sandstone : White-pink, poorly sorted, thickness 1.6'		
			Sandstone : Yellow, medium-grained, well sorted, porous, friable, thickness 3.9'		
			Sandstone : Red, medium-grained, moderately well sorted, porous, friable, thickness 11.8'		
Early Devonian	Tawil Formation				Sub-Unayzah unconformity. Sandstone : White, fine-grained.

Figure 1: Surface type section of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation at latitude 26° 52' 17.4" longitude 43° 36' 18"

The specific heat capacity can be scaled as

$$S_w = \left[ \frac{SHC^{\frac{1}{2}}}{SHC_{max}^{\frac{1}{2}}} \right]^{[3-Df]} \quad [1]$$

Where  $S_w$  the water saturation,  $SHC$  the specific heat capacity in Joule / Kelvin \* kilo gram,  $SHC_{max}$  the maximum specific heat capacity in Joule / Kelvin \* kilo gram, and  $DF$  the fractal dimension. Equation 1 can be proofed from

$$Q = \Delta T * m * SHC \quad [2]$$

Where  $Q$  the heat in Joule,  $\Delta T$  temperature difference in Kelvin,  $m$  the mass in kilo gram,  $SHC$  the specific heat capacity in Joule / Kelvin \* kilo gram. The mass  $m$  can be scaled as

$$m = \left[ \frac{F}{g} \right] \quad [3]$$

Where  $m$  the mass in kilo gram,  $F$  the force in Newton,  $g$  acceleration in meter / square second Insert equation 3 into equation 2

$$Q = \left[ \frac{\Delta T * F * SHC}{g} \right] \quad [4]$$

The acceleration  $g$  can be scaled as

$$g = \left[ \frac{E}{\Psi} \right] \quad [5]$$

Where  $g$  the acceleration in meter / square second,  $E$  the electric field in volt / meter,  $\Psi$  the electric transfer function in volt \* square second / square meter. Insert equation 5 into equation 4

$$Q = \left[ \frac{\Delta T * F * SHC * \Psi}{E} \right] \quad [6]$$

The electric field  $E$  can be scaled as

$$E = \left[ \frac{V}{CEK} \right] \quad [7]$$

Where  $E$  the electric field in volt / meter,  $V$  the velocity in meter / second,  $CEK$  the electro kinetic coefficient in ampere / pascal \* meter. Insert equation 7 into equation 6

$$Q = \left[ \frac{\Delta T * F * SHC * \Psi * CEK}{V} \right] \quad [8]$$

The velocity  $V$  can be scaled as

$$V = \left[ \frac{Q'}{A} \right] \quad [9]$$

Where  $V$  the velocity in meter / second,  $Q'$  the flow rate in cubic meter / second,  $A$  the area in square meter Insert equation 9 into equation 8

$$Q = \left[ \frac{\Delta T * F * SHC * \Psi * CEK * A}{Q'} \right] \quad [10]$$

Equation 10 after rearrange will become

$$Q * Q' = \Delta T * F * SHC * \Psi * CEK * A \quad [11]$$

The flow rate  $Q'$  can be scaled as

$$Q' = \left[ \frac{k * A * \Delta P}{\mu * L} \right] \quad [12]$$

Where  $Q'$  the flow rate in cubic meter / second,  $k$  the permeability in square meter,  $A$  the area in square meter,  $\Delta P$  the differential pressure in Pascal,  $\mu$  the fluid viscosity in Pascal second,  $L$  the capillary length in meter. Insert equation 12 into equation 11

$$Q * k * A * \Delta P = \Delta T * F * SHC * \Psi * CEK * A * \mu * L \quad [13]$$

The maximum permeability  $k_{max}$  can be scaled as

$$Q * k_{max} * A * \Delta P = \Delta T * F * SHC_{max} * \Psi * CEK * A * \mu * L \quad [14]$$

Divide equation 13 by equation 14

$$\left[ \frac{Q * k * A * \Delta P}{Q * k_{max} * A * \Delta P} \right] = \left[ \frac{\Delta T * F * SHC * \Psi * CEK * A * \mu * L}{\Delta T * F * SHC_{max} * \Psi * CEK * A * \mu * L} \right] \quad [15]$$

Equation 15 after simplification will become

$$\left[ \frac{k}{k_{max}} \right] = \left[ \frac{SHC}{SHC_{max}} \right] \quad [16]$$

Take the square root of equation 16

$$\sqrt{\left[ \frac{k}{k_{max}} \right]} = \sqrt{\left[ \frac{SHC}{SHC_{max}} \right]} \quad [17]$$

Equation 17 after simplification will become

$$\left[ \frac{k^{\frac{1}{2}}}{k_{max}^{\frac{1}{2}}} \right] = \left[ \frac{SHC^{\frac{1}{2}}}{SHC_{max}^{\frac{1}{2}}} \right] \quad [18]$$

$$\text{But; } \left[ \frac{k^{\frac{1}{2}}}{k_{max}^{\frac{1}{2}}} \right] = \left[ \frac{SHC^{\frac{1}{2}}}{SHC_{max}^{\frac{1}{2}}} \right] = \left[ \frac{r}{r_{max}} \right] \quad [19]$$

Where  $r$  the pore radius in meter,  $r_{max}$  the maximum pore radius in meter, Take the logarithm of equation 19

$$\log \left[ \frac{k^{\frac{1}{2}}}{k_{max}^{\frac{1}{2}}} \right] = \log \left[ \frac{SHC^{\frac{1}{2}}}{SHC_{max}^{\frac{1}{2}}} \right] = \log \left[ \frac{r}{r_{max}} \right] \quad [20]$$

$$\text{But; } \log \left[ \frac{r}{r_{max}} \right] = \left[ \frac{\log S_w}{3 - Df} \right] \quad [21]$$

Insert equation 21 into equation 20

$$\log[\text{SHC}^{1/2} / \text{SHC}_{\text{max}}^{1/2}] = [\log S_w / 3 - \text{DF}] \quad [22]$$

Equation 22 after log removal will become

$$S_w = \left[ \frac{\text{SHC}^{\frac{1}{2}}}{\text{SHC}_{\text{max}}^{\frac{1}{2}}} \right]^{[3-\text{Df}]} \quad [23]$$

Equation 23 the proof of equation 1 which relates the water saturation, specific heat capacity, maximum specific heat capacity and the fractal dimension. The capillary pressure can be scaled as

$$S_w = [\text{Df} - 3] * \text{Pc} * \text{constant} \quad [24]$$

Where  $S_w$  the water saturation,  $P_c$  the capillary pressure and  $D_f$  the fractal dimension.

## Results and Discussion

Based on field observation the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation were divided here into three units as described in Figure 1. These units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their attained results of the Specific heat capacity fractal dimension and capillary pressure fractal dimension are shown in Table 1. Based on the achieved results it was found that the Specific heat capacity fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 allocated to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as shown in Table 1. The Specific heat capacity fractal dimension and capillary pressure fractal dimension were detected to increase with increasing permeability as proofed in Table 1 owing to the possibility of having interconnected channels. Altogether, additional evidence to proof permeability increases with interconnected channels is the degree of sorting and friability.

**Table 1:** Petro physical model showing the three Shajara Reservoir Units with their corresponding values of Specific heat capacity fractal dimension and capillary pressure fractal dimension

Formation	Reservoir	Sample	Porosity %	K (md)	Grain size	Positive slope of the first procedure Slope=3-Df	Negative slope of the second procedure Slope=Df-3	Specific heat capacity fractal dimension	Capillary pressure fractal dimension
Permo-Carboniferous Shajara Formation	Upper Shajara Reservoir	SJ13	25	973	Coarse-grained	0.2128	-0.2128	2.7872	2.7872
		SJ12	28	1440	Very coarse grained	0.2141	-0.2141	2.7859	2.7859
		SJ11	36	1197	Medium-grained	0.2414	-0.2414	2.7586	2.7586
	Middle Shajara Reservoir	SJ9	31	1394	Medium-grained	0.2214	-0.2214	2.7786	2.7786
		SJ8	32	1344	Medium-grained	0.2248	-0.2248	2.7752	2.7752
		SJ7	35	1472	Coarse-grained	0.2317	-0.2317	2.7683	2.7683
	Lower Shajara Reservoir	SJ4	30	176	Coarse-grained	0.3157	-0.3157	2.6843	2.6843
		SJ3	34	56	Fine-grained	0.5621	-0.5621	2.4379	2.4379
		SJ2	35	1955	Medium-grained	0.2252	-0.2252	2.7748	2.7748
		SJ1	29	1680	Medium-grained	0.2141	-0.2141	2.7859	2.7859

The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were carefully chosen for capillary pressure measurement as proven in Table 1. Their positive slopes of the first procedure log of the Specific heat capacity to maximum Specific heat capacity versus log wetting phase saturation ( $S_w$ ) and negative slopes of the second procedure log capillary pressure ( $P_c$ ) versus log wetting phase saturation ( $S_w$ ) are clarified in Figure 2, Figure 3, Figure 4, Figure 5 and Table 1. Their Specific heat capacity fractal dimension and capillary pressure fractal dimension values are revealed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was described from 1955 md to 56 md which reflects decrease in Specific heat capacity fractal

dimension from 2.7748 to 2.4379 as quantified in table 1. Again, an increase in grain size and permeability was proved from sample SJ4 whose Specific heat capacity fractal dimension and capillary pressure fractal dimension was found to be 2.6843 as described in Table 1.

In contrast, the Middle Shajara reservoir which is separated from the Lower Shajara reservoir by an unconformity surface as revealed in Figure 1. It was nominated by four samples (Figure 1), three of which named as SJ7, SJ8, and SJ9 as illuminated in Table 1 were chosen for capillary measurements as described in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are shown in Figure 6, Figure 7

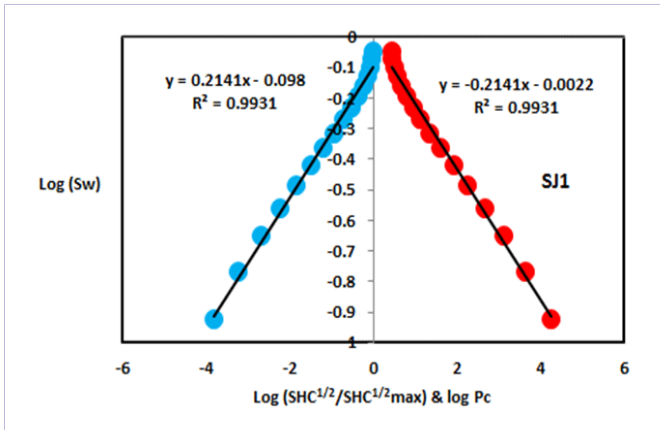


Figure 2: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ1

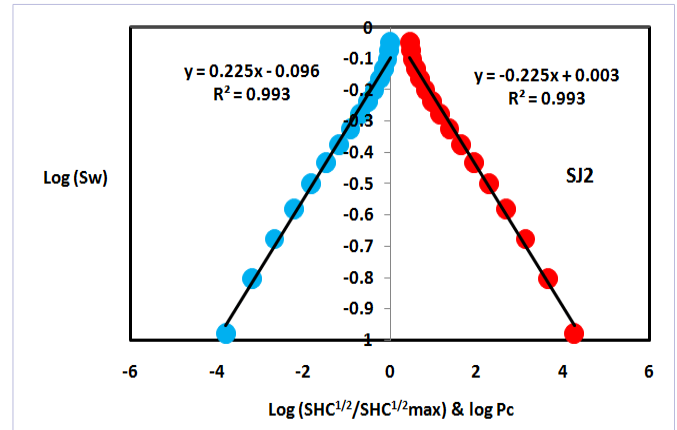


Figure 3: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ2

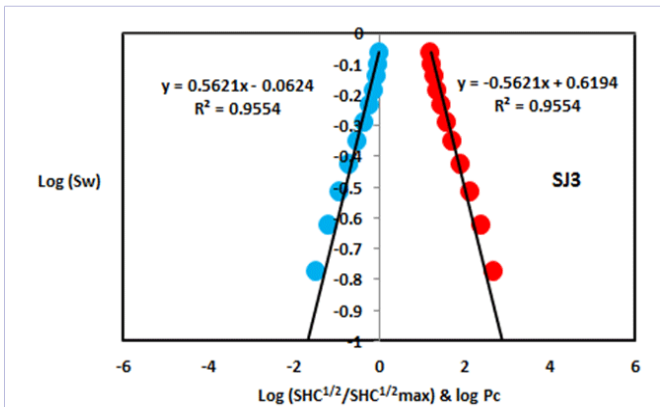


Figure 4: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ3

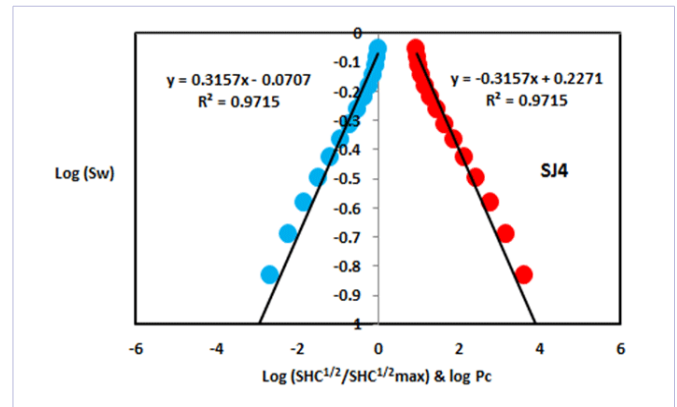


Figure 5: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ4

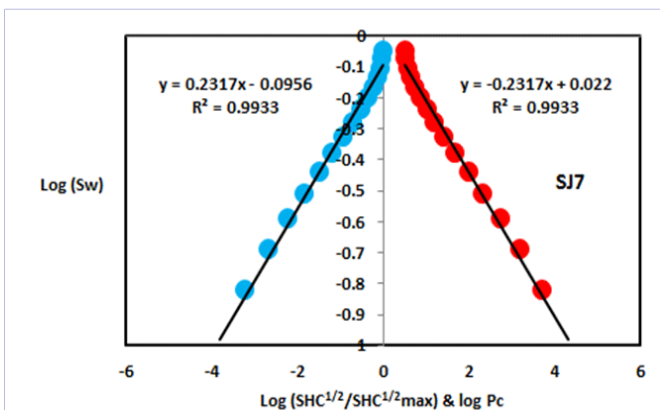


Figure 6: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ7

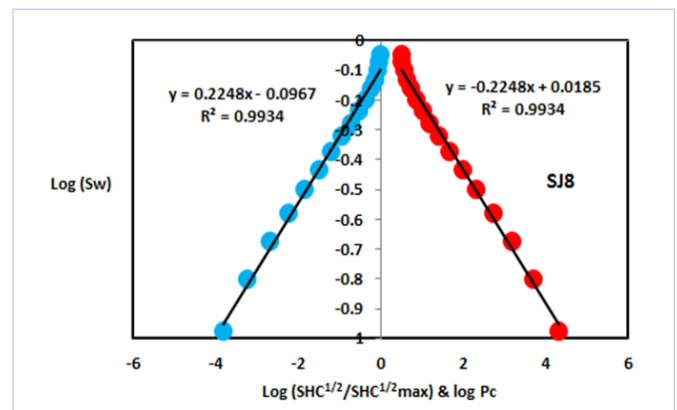


Figure 7: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ8

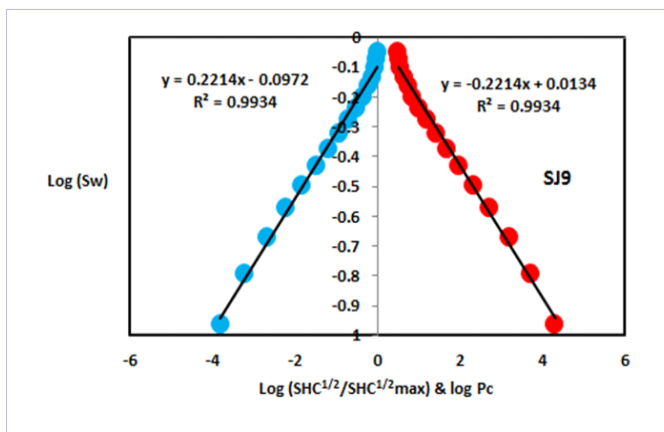


Figure 8: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ9

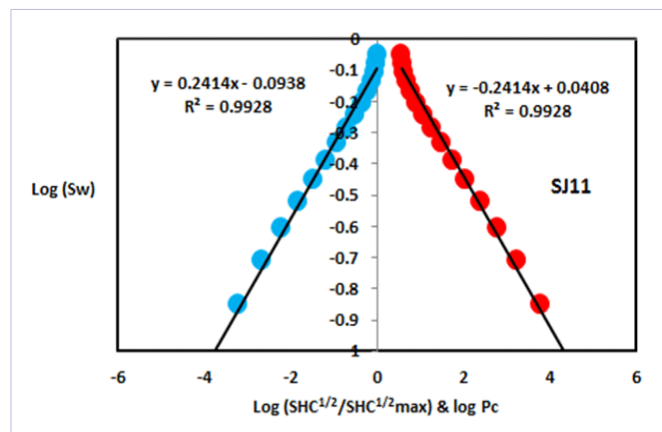


Figure 9: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ11

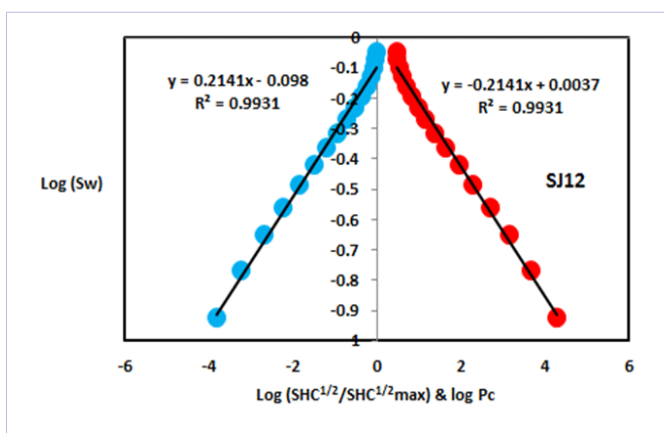


Figure 10: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ12

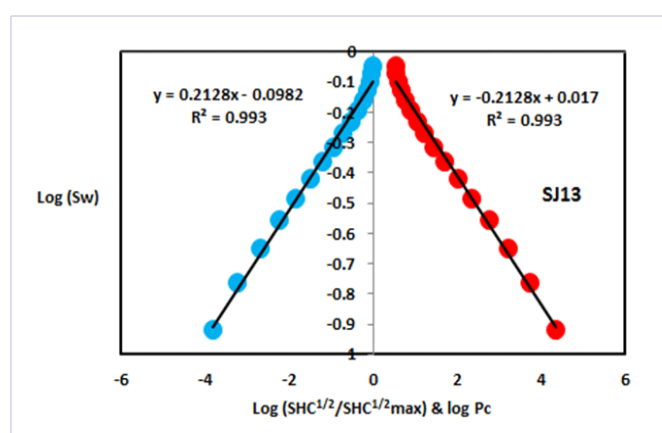


Figure 11: Log (SHC<sup>1/2</sup>/SHC<sup>1/2max</sup>) & log pc versus log Sw for sample SJ13

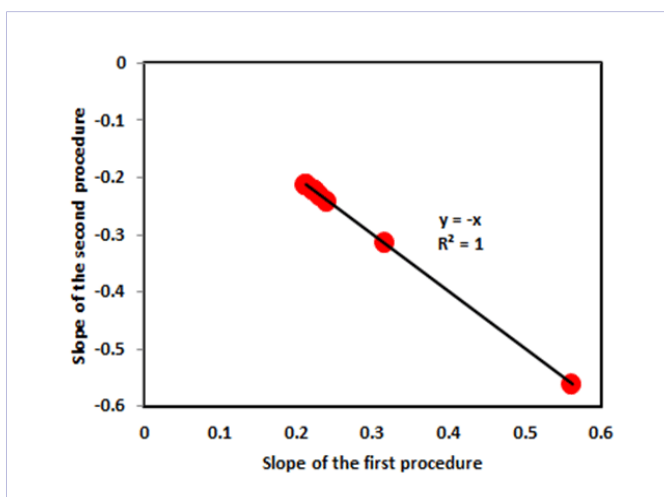


Figure 12: Slope of the first procedure versus slope of the second procedure

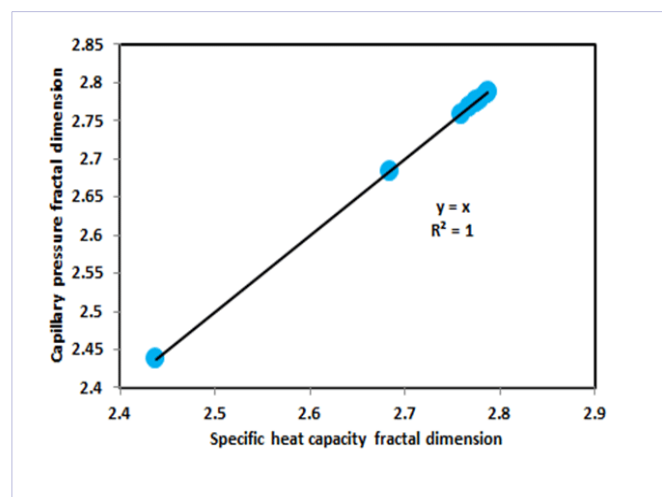


Figure 13: Specific heat capacity fractal dimension versus capillary pressure fractal dimension

and Figure 8 and Table 1. Furthermore, their Specific heat capacity fractal dimensions and capillary pressure fractal dimensions show similarities as defined in Table 1. Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in table 1.

On the other hand, the Upper Shajara reservoir was separated from the Middle Shajara reservoir by yellow green mudstone as shown in Figure 1. It is defined by three samples so called SJ11, SJ12, and SJ13 as explained in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in Figure 9, Figure 10 and Figure 11 and Table 1. Moreover, their Specific heat capacity fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as simplified in table 1.

Overall a plot of positive slope of the first procedure versus negative slope of the second procedure as described in Figure 12 reveals three permeable zones of varying Petro physical properties. These reservoir zones were also confirmed by plotting Specific heat capacity fractal dimension versus capillary pressure fractal dimension as described in Figure 13. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment.

## Conclusion

The sandstones of the Shajara Reservoirs of the permo-Carboniferous Shajara Formation were divided here into three units based on Specific heat capacity fractal dimension. The Units from base to top are: Lower Shajara Specific Heat Capacity Fractal Dimension Unit, Middle Shajara Specific Heat Capacity Fractal Dimension Unit, and Upper Shajara Specific Heat Capacity Fractal Dimension Unit. These units were also proved by capillary pressure fractal dimension. As a whole, the fractal dimension was found to increase with increasing grain size, degree of sorting, friability, and permeability owing to possibility of having interconnected channels.

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