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On Super Edge-Magicness of Double Star

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Abstract

For a graph G, a bijection $f: V(G)\cup E(G)\to\{1,2,3,\ldots,p+q\}$ is called an edge-magic labeling of G if there exists a constant s such that $f(u)+f(uv)=s$, for any edge (u,v) *of* G. f is said to be super edge-magic if $f(V(G))=\{1,2,3,\ldots,p\}$. After investigating the super edge-magicness of double stat $S_{m,n}$, a partial solution to a research problem ("For what m and n does $S_{m,n}$ have an edge magic labeling?, stated by Marr and Wallis in [6]") was given in this paper

Keywords: Graph labeling; Edge-magic graphs; Super edge-magic graphs;

AMS Mathematics Subject Classification: 05C78.

Introduction

In this paper we consider only finite and simple undirected graphs. The vertex and edge sets of a graph G are denoted by V(G) and E(G) respectively and we let |V(G)| = p and |E(G)| = q. For graph theoretic notations, we follow [1,2]. A labeling of a graph G is a mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [4].

In 1998, Enomoto, et al. [3] introduced the concept of super edge-magic graphs. In 2005, Sugeng and Xie, [7] constructed some super edge-magic total graphs. The usage of the word "super" was introduced in [4]. A edge-magic total labeling is a bijection f from $V(G)\cup E(G)$ to the integers {1,2,...,p+q} with the property that for every edge f(u) + f(v) + f(uv) = s for some constant s, such a labeling is (V)-super if f(V(G)) = {1,2,...,p}. A graph G is called edge-magic (resp. super edge-magic) if there exists an edge-magic (resp. super edge-magic) labeling of G.

Recently, Marimuthu and Balakrishnan, [5], introduced the notion of super edge-magic graceful graphs to solve some kind of network problems. A (p, q) graph G with p vertices and q edges is edge magic graceful if there exists a bijection f : $V(G)\cup E(G)\to\{1,2,\ldots,p+q\}$ such that $|f(u)+f(v)-f(uv)|$ = k, a constant for any edge uv of G. G is said to be super edge-magic graceful if f(V(G)) = {1,2,...,p}.

The double star $S_{m,n}$ has two adjacent central vertices x and y. There are m leaves(vertices) $x_1,x_2,\ldots,x_m$ adjacent to x and n leaves(vertices) $y_1,y_2,\ldots,y_n$ adjacent to y. A (super) edge-magic total labeling of this graph can be specified by the list $(\{f(x_1),f(x_2),\ldots,f(x_m)\},f(x),f(y),\{f(y_1),f(y_2),\ldots,f(y_n)\})$. One solution for to be edge-magic is ({8,11},2,5, {4,10}) with s = 16. One solution for $S_{2,2}$ to be super edge-magic is ({6,3},2,5, {4,1}) with s = 16.

Some existing results found in [6]

i. All Paths have edge-magic total labeling.

ii. A Cycle C_n is super edge-magic if and only if n is odd.

iii. The Complete bipartite graph $K_{m,n}$ is super edge-magic if and only if $n=1$ or $m=1$.

In [6], Marr and Wallis quoted a research problem "For what m and n does $S_{m,n}$ have an edge magic labeling?". In this paper, we prove the following

i. $S_{n,n+2}$ is super edge-magic

ii. $S_{n,n}$ is super edge-magic if n is even.

Main Result

In this section, we prove two main theorems.

Theorem 4.1: The double star $S_{n,n+2}$ is super edge-magic.

Proof: Let $G\cong S_{n,n+2}$ has two adjacent central vertices x and y with n vertices $x_1,x_2,\ldots,x_n$ adjacent to x and $n+2$ vertices $y_1,y_2,\ldots,y_{n+2}$ adjacent to y. Clearly $p=2n+4$ and $q=2n+3$. Define a total labeling $f: V(G)\cup E(G)\to\{1,2,\ldots,p+q\}$ by $f(x)=2$; $f(y)=4$; $f(y_i)=2i-1$ and $f(x_j)=2j+4$, for all $1\le i\le n+2$ *and* $1\le j\le n$.

Now the edges of G can be labeled as shown in Table 1 and Table 2.

Table 1: The edge label of $S_{n,n+2}$ whose $n+2$ edges are labeled

f	y_1	y_2	$\cdots$	y_i	$\cdots$	y_{n+1}	y_{n+2}
y	$4n+7$	$4n+5$		$4n-(2i-9)$		$2n+7$	$2n+5$

Table 2: The edge label of $S_{n,n+2}$ whose n edges are labeled

f	y_1	y_2	$\cdots$	y_i	$\cdots$	y_{n+1}	y_{n+2}
x	$4n+4$	$4n+2$		$4n-(2i-6)$		$2n+8$	$2n+6$

And $f(xy)=4n+6$. It is easily seen that f is super edge-magic labeling with magic constant $s=4n+12$. Hence the graph $S_{n,n+2}$ is super edge-magic. (Figure 1)

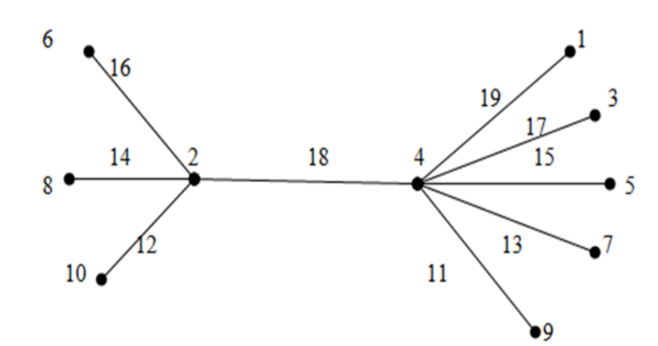

Figure 1: Super edge-magic labeling of $S_{3,5}$ with magic constant $s=24$

Theorem 4.2: The double star $S_{n,n}$ is super edge-magic if n is even.

Proof: Let $G \cong S_{n,n}$ has two adjacent central vertices x and y with n vertices

$x_1, x_2, \ldots, x_n$ adjacent to x and n vertices $y_1, y_2, \ldots, y_n$ adjacent to y. Clearly $p=2n+4$ and $q=2n+3$. Define a total labeling $f: V(G)\cup E(G) \to \{1,2,\ldots,p+q\}$ by

$$f(y_i)=\begin{cases} i & if \quad 1\le i\le \frac{n}{2} \\ \left(\frac{n}{2}+1\right)+i & if \quad \frac{n}{2}+1\le i\le n \end{cases}$$

$$f(x_i)=\begin{cases} \left(\frac{n}{2}+1\right)+i & if \quad 1\le i\le \frac{n}{2} \\ (n+2)+i & if \quad \frac{n}{2}+1\le i\le n \end{cases}$$

$f(x)=\left(\frac{n}{2}+1\right)$ and $f(y)=\left(\frac{n}{2}+1\right)+(n+1)$.

Now the edges of G can be labeled as shown in Table 3 and Table 4.

Table 3: The edge label of $S_{n,n}$ whose first n edges are labeled

f	y
y_1	$(3n+3)+\left(\frac{n}{2}\right)$
y_2	$(3n+3)+\left(\frac{n}{2}-1\right)$
$\vdots$	$\vdots$
y_j	$(3n+3)+\left(\frac{n}{2}+1-j\right)$
$\vdots$	$\vdots$
$y_{\frac{n}{2}}$	$(3n+3)+1$
$y_{\frac{n}{2}+1}$	$(2n+2)+\left(\frac{n}{2}\right)$
$\vdots$	$\vdots$
y_k	$(2n+2)+(n+1-k)$
$\vdots$	$\vdots$
y_{n-1}	$(2n+2)+2$
y_n	$(2n+2)+1$

Table 4: The edge label of $S_{n,n}$ whose second n edges are labeled

f	x
x_1	$\left(\frac{7n}{2}+3\right)+\left(\frac{n}{2}\right)$
x_2	$\left(\frac{7n}{2}+3\right)+\left(\frac{n}{2}-1\right)$
$\vdots$	$\vdots$
x_j	$\left(\frac{7n}{2}+3\right)+\left(\frac{n}{2}+1-j\right)$
$\vdots$	$\vdots$
$x_{\frac{n}{2}}$	$\left(\frac{7n}{2}+3\right)+1$
$x_{\frac{n}{2}+1}$	$\left(\frac{5n}{2}+2\right)+\left(\frac{n}{2}\right)$
$\vdots$	$\vdots$
x_k	$\left(\frac{5n}{2}+2\right)+(n+1-k)$
$\vdots$	$\vdots$
x_{n-1}	$\left(\frac{5n}{2}+2\right)+2$
x_n	$\left(\frac{5n}{2}+2\right)+1$

$f(xy)=3n+3$ It is easily seen that f is super edge-magic labeling with magic constant $s=5n+6$. Hence the graph $S_{n,n}$ is super edge-magic. (Figure 2)

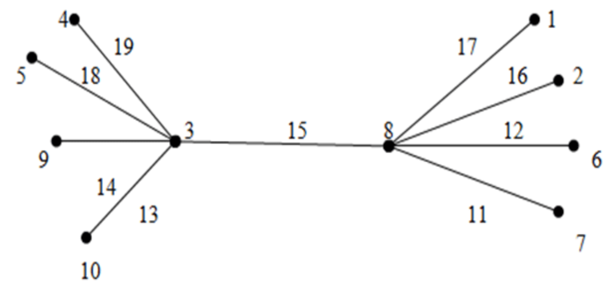

Figure 2: Super edge-magic labeling of $S_{4,4}$ with magic constant $s=24$

Conclusion

In this paper, we had given two different solutions to a Research Problem given by Marr and Wallis in [6].

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