

Non-Synchronous Sensor Network Positioning Method Based On RSS and TOA Hybrid Measurement

Lu Kangli^{1*}, Wu Xiaoping², LvYangmin³ and Hua Yuting⁴

¹School of Information Engineering, Zhejiang Agriculture and Forestry University, Lin'an 311300, China

²Zhejiang Key Laboratory of Forestry Intelligent Monitoring and Information Technology, Lin'an 311300, China

³School of Information Engineering, Zhejiang Agriculture and Forestry University, Lin'an 311300, China

⁴School of Information Engineering, Zhejiang Agriculture and Forestry University, Lin'an 311300, China

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*Corresponding author: Lu Kangli, School of Information Engineering, Zhejiang Agriculture and Forestry University, Lin'an 311300, China, E-mail: 1215446881@qq.com

Abstract

The method of single range-based measurement will lead to an increase in the uncertainty of positioning results and a lower positioning accuracy. To deal with the positioning problem in three dimensional non-synchronous sensor networks, an accurate positioning method is proposed by using the hybrid Received Signal Strength (RSS) and Time-of-Arrival (TOA) measurements. The Unconstrained Linear Least Squares (ULLS) and Constrained Linear Squares (CLLS) methods are put forward to obtain the node position coordinates by converting the non-linear optimization model established by hybrid measurements into linear equations. The accuracy of the designed algorithm was tested by simulation, and the influence of different measurement noise on the estimation error of the hybrid measurement method was analyzed. The results show that the hybrid measurement positioning method has less error than the single measurement method. The constrained CLLS is more accurate than the unconstrained ULLS. The CLLS and ULLS linear estimation methods have high stability and positioning accuracy under low noise conditions.

Keywords: Positioning; Time synchronization; Received signal strength; Hybrid measurement;

Introduction

The Wireless Sensor Networks (WSN) use a large number of static or mobile wireless transmission nodes for sensing and collection. They have broad application prospects in the fields of environmental monitoring, target tracking, military and intelligent transportation systems, and have been concerned and explored by many researchers [1,2]. The implementation of functions such as target tracking, geographical location routing, and fault alarm in sensor network applications is based on node location [3,4]. Node location technology is one of the basic support technologies of wireless sensor networks [5,6]. Sensor network positioning technology has become a key factor in its application promotion, and the node positioning technology with

superior development performance has become an important research hotspot of sensor network positioning content. In a real scenario, the sensor network area deployed by nodes is usually in a 3D environment. However, most of the existing localization algorithms are researched and optimized based on 2D space, which is different from the real environment, resulting in the inability to locate specific locations with high precision. Therefore, the research of 3D positioning algorithm is one of the key contents to solve the problem of wireless sensor networks node positioning in the future [7].

The use of known location beacon nodes to estimate the unknown node position coordinates is a common sensor network positioning method, which relies on some ranging methods between nodes, such as Time of Arrival (TOA) and Arrival Time Difference (TDOA), Angle of Arrival (AOA) and Signal Reception Strength (RSS) [8-11]. Since each node uses a separate clock module for timing, the clock module timing will drift with time and environmental parameters, so the timing of each clock module is not synchronized, which is called the asynchronous sensor network [12]. The single-time measurement method can solve the node position estimation of the asynchronous sensor network, but the single measurement method leads to the increase of uncertainty of the positioning result, the unreliable positioning result, and the low positioning accuracy, it is more difficult to apply to the positioning problem of the 3D sensor network.

The time measurement implementation principle is relatively simple. For example, the literature [13] uses multiple antenna receivers, and proposes a joint estimation method of node clock deviation and target position, which realizes the joint calculation of time synchronization and positioning. And the literature [14] introduced a hybrid measurement method combining TOA and AOA. The RSS measurement does not require additional hardware facilities, and the measurement cost is low, there is also a hybrid

measurement method of RSS and TDOA proposed in literature [15]. Among the existing three-dimensional positioning methods, some are derived from three-dimensional positioning methods in three-dimensional space, and some propose new positioning algorithms. For example, the literature [16] solving the problem of 3D location based on RSS and AOA measurements. Therefore, this paper introduces a hybrid RSS and TOA measurement method to solve the positioning problem in the 3D scene in the asynchronous sensor network.

With the increasing demand for location service applications, the positioning accuracy requirements are getting higher and higher. How to reduce the positioning error of positioning algorithm design has become a research hotspot in this field. By establishing a distance-constrained optimization model between nodes, researchers have continuously proposed positioning algorithms for sensor networks, such as Maximum Likelihood (ML) estimation, linear algebra method and convex optimization, and so on [17-19]. The numerical calculation method of ML estimation method relies on the selection of the initial solution, and has better precision in weak noise environment, but it may fall into local optimum. For this reason, linear algebra method and convex optimization method are proposed. The convex optimization method (including semi-definite programming, quadratic cone planning, etc.) relaxes the optimization model into a convex optimization problem, which is a popular method in the current sensor network positioning method, but the convex optimization function has more variables and equality constraint, and the computational complexity is high. Because of the relaxation, the estimation result is not optimal. In order to reduce the computational complexity, the linear algebra method directly represents the calculation result as an algebraic solution, which is faster in calculation and avoids the trouble caused by the initial value selection.

The TOA ranging method is used to establish the distance relationship between nodes by measuring the arrival time of the signal. The accuracy of the ranging depends on the accurate timing ability of the node clock. However, due to the change of environmental factors, may lead to the asynchrony of the timing clock of the node. In this paper, based on the chronograph clock model of signal transmission between nodes, the relationship between observation time and actual time is derived, and the joint estimation equation of time synchronization and positioning is established. Based on the node clock drift and deviation model, a joint linear estimation method of time synchronization and node localization is proposed to realize simultaneous estimation of node clock drift rate, deviation and position coordinates. The nodes communicate with each other in the form of electromagnetic waves, and the Received Signal Strength (RSS) is attenuated with the extension of the transmission path. The RSS method does not need additional hardware, is simple to implement, it has characteristics of low power consumption, low cost, and the like, and is widely used. The sensor network node positioning is realized by the RSS and TOA hybrid measurement between the nodes, which can make the positioning result more reliable and the positioning accuracy

higher. Since the unknown parameters of the 3D space are more than the 2D plane, the difficulty is also increased. The positioning method using the hybrid measurement is more reliable and accurate than the single ranging method, and is more suitable for the positioning problem of the asynchronous sensor network. Based on the RSS and TOA hybrid measurement technique between nodes, this paper proposes a precise positioning method for the wireless signal strength (RSS) and Time of Arrival (TOA) hybrid measurement of the 3D asynchronous sensor network. The nonlinear optimization model established by the hybrid measurement is transformed into a linear equation, and the Unconstrained Linear Least Squares (ULLS) and Constrained Linear Singularity (CLLS) methods for node position coordinate estimation are proposed respectively, and compared with the model of the Carmer-Rao Lower Bound(CRLB) values were compared. The design of hybrid measurement method for 3D spatial positioning depends on less number of beacon nodes and high positioning accuracy.

The first part of this paper begins with introduced the problem description of hybrid positioning of RSS and TOA; the second part deduces the calculation method of ULLS and CLLS; the third part deduces the lower bound of carmer-Rao (CRLB) of model; the fourth part is simulation and analysis; the last part is the conclusion.

Problem Description

The coordinates of unknown nodes are calculated in two-dimensional plane. Suppose that there are N beacon nodes with known position coordinates, and their coordinate positions are respectively $\mathbf{x}_i = [x_i \ y_i]^T$ ($i=1,2,\dots,N$). At the same time, there is an unknown node to be located in the area, and the position coordinate is assumed to be $\mathbf{x}=[x \ y]^T$. Using the ranging method to obtain the coordinates of the unknown node, it is obvious that this ignores the gradient difference between the unknown node and the beacon node, and it is unrealistic. The coordinate parameters of the spatial position of the 3D sensor network space are increased compared with the 2D plane, including three directions of x, y, and z. Increasing the gradient of Z axis can make more accurate calculation. Assuming that the coordinate positions of the beacon a node of the N known position coordinates are respectively $\mathbf{x}_i = [x_i \ y_i \ z_i]^T$ ($i=1,2,\dots,N$). At the same time, the location coordinates of the unknown node to be located in the area is assumed to be $\mathbf{x}=[x \ y \ z]^T$. (Figure 1)

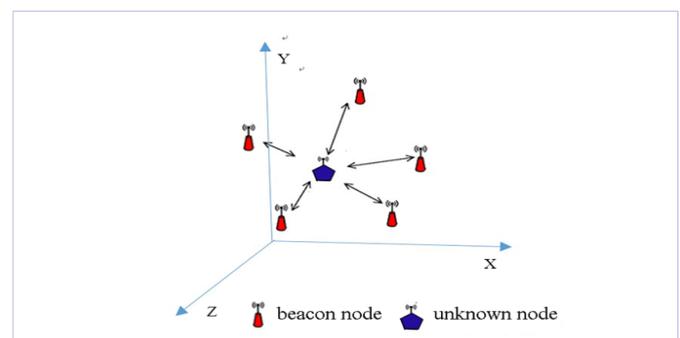


Figure 1: Schematic diagram of unknown node and beacon node model

To estimate the location of an unknown node, the Time of Arrival (TOA) of the signal is measured to represent the spatial distance between the unknown node and the beacon nodes. However, in the non-synchronous sensor network, the node timing clock module is not synchronized due to environmental impact and other reasons, resulting in the node observation time being inconsistent with the actual real time. In order to improve the accuracy of the measurement results, this paper uses the drift and deviation clock timing model to represent the relationship between observation time and actual real time. ω and θ respectively represent the drift rate and the deviation amount of the clock, and represent the time change rate and the time difference between the observation time and the actual real time on the unknown node. The observation time T and the actual real time t of the unknown node are expressed as a relation:

$$T = \omega t + \theta \quad (1)$$

The clock drift rate and the deviation amount of the beacon nodes i are assumed to be ω_i, θ_i . According to the same principle, the observation time and the real time of the beacon nodes i also have the same relationship as equation (1). In this model, it is assumed that the clock parameters of the beacon nodes, including the drift rate ω_i and the deviation amount θ_i are calibrated, and are known parameters, while the unknown node clock parameters ω and θ are unknown parameters.

The unknown node sends a signal to the beacon nodes i at $T_{1,i}$ time (the observation time of the unknown node clock at $T_{1,i}$) and the beacon nodes i receive the signal of the unknown node at $R_{1,i}$ time (the observation time of the beacon nodes i clock at $R_{1,i}$), According to equation (1) and the signal transmission process has the following relationship:

$$\frac{R_{1,i}}{\omega_i} - \frac{T_{1,i}}{\omega} + \frac{\theta}{\omega} - \frac{\theta_i}{\omega_i} = t_i + m_i \quad (2)$$

In equation (2) t_i , is the Signal arrival time between nodes $t_i = d_i/c$, d_i is the spatial distance between the unknown node and the beacon nodes i , c is the propagation speed of the signal, $c = 3 \times 10^8$ m/s m_i represents the time measurement noise between the unknown node and the beacon nodes i . It can be assumed that m_i obeys the Gaussian distribution with a mean of zero, and a variance of $\delta_{i,m}^2$, which is denoted as $m_i \in N(0, \delta_{i,m}^2)$.

When the beacon nodes i receive the signal of the unknown node, the beacon nodes i send a signal to the unknown node at $T_{2,i}$ time ($T_{2,i}$ is the observation time of the beacon nodes i clock), and the unknown node at $R_{2,i}$ time ($R_{2,i}$ is the observation time of the unknown node clock) receives the signal of the beacon nodes, a similar derivation process of the equation (2), there is:

$$\frac{R_{2,i}}{\omega} - \frac{T_{2,i}}{\omega_i} + \frac{\theta_i}{\omega_i} - \frac{\theta}{\omega} = t_i + n_i \quad (3)$$

In the formula, the definition of t_i is the same as equation (2), and n_i is also the noise part. It is assumed that n_i obeys the mean of zero, and the Gaussian distribution of variance $\delta_{i,n}^2$ is, recorded as $n_i \in N(0, \delta_{i,n}^2)$.

It can be seen from equations (2) and (3) that the clock parameters and the position parameters are intertwined, and the clock parameter estimation and positioning have commonality in time characteristics. For this reason, the clock parameter estimation and the position parameters can be jointly estimated, and carried out to realize both the clock parameters estimation and the position estimation. Since the positional parameter in the 3D sensor network space has more z-direction than the 2D plane, the method of estimating the position coordinates of the unknown node only by measuring the arrival time of the signal between the unknown node and the beacon nodes, and the measurement result is not accurate. So the (RSS) positioning method is added to measure the signal strength of unknown node and beacon nodes. Express the RSS measurement value between the unknown node and the beacon node i as p_i . According to the logarithmic attenuation model of RSS ranging, there is the following RSS relation expression:

$$p_i = p_o - 10\beta \lg d_i + \varepsilon_i \quad (4)$$

In equation (4), β is the path attenuation index, The β value can be obtained through experimental analysis in advance, and the value varies with the change of the propagation medium, generally between 2 and 5. p_o is the signal emission intensity of the unknown node, which is related to the antenna gain of the node and the battery supply. p_i is the signal reception strength of the beacon nodes i . d_i is the spatial linear distance between the unknown node to be located and the beacon nodes i , $d_i = \|\mathbf{x} - \mathbf{x}_i\|$, ε_i represents the measurement noise between the unknown node and the beacon nodes i , and it can be assumed that ε_i obeys the Gaussian distribution with a mean of zero, and variance is $\delta_{i,\varepsilon}^2$, which is recorded as $\varepsilon_i \in N(0, \delta_{i,\varepsilon}^2)$.

To estimate the unknown node position coordinate \mathbf{x} , with TOA and RSS hybrid measurements as known values, and the maximum likelihood (ML) estimates by optimizing the following expressions:

$$\arg \min_{\mathbf{x}} \sum_{i=1}^N \left(\frac{1}{\delta_{i,m}^2} r_{i,m}^2 + \frac{1}{\delta_{i,n}^2} r_{i,n}^2 + \frac{1}{\delta_{i,\varepsilon}^2} r_{i,\varepsilon}^2 \right) \quad (5)$$

In equation (5), $r_{i,m}$, $r_{i,n}$, $r_{i,\varepsilon}$, respectively represent the error items of the two arrival time measurements and the wireless received signal strength measurement.

$$r_{i,m} = \frac{R_{1,i}}{\omega_i} - \frac{T_{1,i}}{\omega} + \frac{\theta}{\omega} - \frac{\theta_i}{\omega_i} - t_i, \quad r_{i,n} = \frac{R_{2,i}}{\omega} - \frac{T_{2,i}}{\omega_i} + \frac{\theta_i}{\omega_i} - \frac{\theta}{\omega} - t_i,$$

$r_{i,\varepsilon} = p_i - p_o + 10\beta \lg d_i$ The optimization function established by equation (5) is a nonlinear equation, and its numerical calculation method depends on the selection of the initial value. If the initial value selection is not suitable, it may fall into local optimum, which causes serious deviation of the positioning result. Therefore, this paper converts the nonlinear optimization equation into a linear equation, and the two-step calculation method is used to accurately calculate the position coordinates and clock parameters of the unknown node.

Linear Least Squares Estimation Method

Considering the approximate linearization of the measurement equation under small noise conditions, the nonlinear optimization model described in equation (5) is transformed into a linear equation. The designed calculation method is divided into two steps: the Unconstrained Linear Least Squares (ULLS) method and the Constrained Linear Least Squares (CLLS) method.

Unconstrained Least Squares Method

Adding equations (2) and (3) to eliminate common items:

$$\frac{R_{2,i} - T_{1,i}}{\omega} + \frac{R_{1,i} - T_{2,i}}{\omega_i} = 2t_i + m_i + n_i \quad (6)$$

Since ω is very close to 1, it can be assumed that $\omega = \frac{1}{1+\delta}$, δ is a variable close to zero, so equation (6) can be rewritten as:

$$\mu_i \delta + \lambda_i = 2t_i + m_i + n_i \quad (7)$$

In equation (7), $\lambda_i = \frac{R_{1,i} - T_{2,i}}{\omega_i} + R_{2,i} - T_{1,i}$, $\mu_i = R_{2,i} - T_{1,i}$ it can be seeing that λ_i and μ_i are known parameters, since $t_i = d_i / c$ and $d_i = |\mathbf{x} - \mathbf{x}_i|$, square sides of the equation (7), ignoring the second high-order term, and sorting the expression:

$$8\mathbf{x}_i \mathbf{x} - 4\mathbf{x}^T \mathbf{x} + 2c^2 \mu_i \lambda_i \delta = 4\mathbf{x}_i^T \mathbf{x}_i - c^2 \lambda_i^2 + c^2 d_i (m_i + n_i) \quad (8)$$

Equation (8) shows the transformed TOA measurement equation, $i = 1, 2, \dots, N$. Let $\mathbf{z} = [\mathbf{x} \ \mathbf{x}^T \mathbf{x} \ \delta]^T$, equation (8) can be written as a linear matrix:

$$\mathbf{A}_1 \mathbf{z} = \mathbf{b}_1 + \gamma_1 \quad (9)$$

In equation (9), the row vectors of matrix \mathbf{A}_1 of dimension $N \times 5$ is $[8\mathbf{x}_i \ -4 \ 2c^2 \mu_i \lambda_i]$, the column vector \mathbf{b}_1 and the row element values of γ_1 of N row are $[4\mathbf{x}_i^T \mathbf{x}_i - c^2 \lambda_i^2]$ and $[c^2 d_i (m_i + n_i)]$ respectively. Equivalent transformation of the RSS measurement equation, that is, equivalent shift transformation of equation (4), rewriting it to:

$$d_i = 10^{\frac{p_0 - p_i + \varepsilon_i}{10\beta}} \quad (10)$$

Considering that in the smaller noise range, the Taylor series expansion is applied to the right side of equation (10), ignore the high order term, and equation (10) is transformed into:

$$d_i = \tau_i + \frac{\tau_i \ln 10}{10\beta} \varepsilon_i \quad (11)$$

In equation (11), $i = 1, 2, \dots, N$, $\tau_i = 10^{\frac{p_0 - p_i}{10\beta}}$ Square the two sides of the equation (11), omitting the high-order terms, it can get:

$$d_i^2 = \tau_i^2 + \frac{\tau_i^2 \ln 10}{5\beta} \varepsilon_i \quad (12)$$

Since $d_i = |\mathbf{x} - \mathbf{x}_i|$, there is an expression for expansion of equation (12):

$$-2\mathbf{x}_i^T \mathbf{x} + \mathbf{x}^T \mathbf{x} = -\mathbf{x}_i^T \mathbf{x}_i + \tau_i^2 + \frac{\tau_i^2 \ln 10}{5\beta} \varepsilon_i \quad (13)$$

Equation (13) shows the transformed RSS measurement equation, $i = 1, 2, \dots, N$, and the same equation (13) can be written as a linear matrix form:

$$\mathbf{A}_2 \mathbf{z} = \mathbf{b}_2 + \gamma_2 \quad (14)$$

In equation (14), the row vector of matrix \mathbf{A}_2 of dimension $N \times 5$ is $[-2\mathbf{x}_i^T \ 1 \ 0]$. The column vector \mathbf{b}_2 and the row element values of γ_2 of N row are $[-\mathbf{x}_i^T \mathbf{x}_i + \tau_i^2]$, $[\frac{\tau_i^2 \ln 10}{5\beta} \varepsilon_i]$ Establish a unified matrix form by combining equations (9) and (14):

$$\mathbf{A} \mathbf{z} = \mathbf{b} + \gamma \quad (15)$$

In equation (15), $\mathbf{A} = [\mathbf{A}_1^T \ \mathbf{A}_2^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T \ \mathbf{b}_2^T]^T$ and $\mathbf{A} \in \mathbb{R}^{2N \times 5}$, $\mathbf{b} \in \mathbb{R}^{2N}$, $\mathbf{v} \in \mathbb{R}^{2N}$.

According to the principle of least squares, the estimated value of parameters \mathbf{z} is:

$$\mathbf{z} = (\mathbf{A}^T \hat{\mathbf{O}}_\gamma^{-1} \mathbf{A})^{-1} \mathbf{A}^T \hat{\mathbf{O}}_\gamma^{-1} \mathbf{b} \quad (16)$$

In equation (16), the matrix $\hat{\mathbf{O}}_\gamma$ with a dimension of $2N \times 2N$ is a weight matrix, whose value is $\hat{\mathbf{O}}_\gamma = \Sigma(\gamma^T \gamma)$. Representing the variance of $\gamma^T \gamma$ and its value is further expressed as:

$$\hat{\mathbf{O}}_\gamma = \begin{bmatrix} \Sigma(\gamma_1^T \gamma_1) & \Sigma(\gamma_1^T \gamma_2) \\ \Sigma(\gamma_2^T \gamma_1) & \Sigma(\gamma_2^T \gamma_2) \end{bmatrix} \quad (17)$$

Because γ_1 and γ_2 are not related, there are $\Sigma(\gamma_1^T \gamma_2)$, $\Sigma(\gamma_2^T \gamma_1)$ equal to zero, will γ_1 , γ_2 are substituted into each equation, the other values of $\hat{\mathbf{O}}_\gamma$ are as follows $\Sigma \gamma$

$$\begin{cases} \Sigma(\gamma_1^T \gamma_1) = \text{diag} \left\{ \frac{\tau_i^4}{4.7\beta^2} \delta_{i,\sigma}^2 \right\} \\ \Sigma(\gamma_2^T \gamma_2) F(\mathbf{v}_2^T \mathbf{v}_2) = \text{diag} \left\{ c^4 d_i^2 (\delta_{i,m} + \delta_{i,n})^2 \right\} \end{cases} \quad (18)$$

Assume the estimated error $\Delta \mathbf{z}$ of the parameter \mathbf{z} , the value is:

$$\Delta \mathbf{z} = (\mathbf{A}^T \hat{\mathbf{O}}_\gamma^{-1} \mathbf{A})^{-1} \mathbf{A}^T \hat{\mathbf{O}}_\gamma^{-1} \gamma \quad (19)$$

The variance of the estimation error $\Delta \mathbf{z}$ is expressed as:

$$\text{cov}(\Delta \mathbf{z}) = (\mathbf{A}^T \hat{\mathbf{O}}_\gamma^{-1} \mathbf{A})^{-1} \quad (20)$$

Extracting $\mathbf{z}(1:3)$ slave from the parameter \mathbf{z} is the position coordinates of the unknown node being located. The above solution process is not considered the mutually constrained relationship between the fourth element and the first three element values in $\mathbf{z} = [\mathbf{x} \ \mathbf{x}^T \mathbf{x} \ \delta]^T$, so the calculation method is called the unconstrained least squares (ULLS) method of the RSS and TOA hybrid positioning problem. Equation (16) obtains an approximate estimate of the coordinates of the unknown node being located, and the exact value of the unknown coordinates of

the unknown node can be calculated using the mutual constraint relation between the vector $\mathbf{z} = [\mathbf{x} \ x^T \ \delta]^T$ elements. Since the fifth element and the first four elements are not constrained, this article only selects the constraint relationship of the first four elements. The clock drift rate ω can be further solved by $\omega = \frac{1}{1+\delta}$, the equation (2) subtract the equation (3) can further calculate the clock offset θ .

Constrained Least Squares

The results obtained by the ULLS method are further optimized by using the constraint relationship of each element value in the vector \mathbf{Z} . Assume that the true value of the coordinate position of the unknown node is $\mathbf{x}^o = [x^o \ y^o \ z^o]^T$, there are the following relationships:

$$\begin{cases} x^{o2} = [\mathbf{z}(1) + \Delta\mathbf{z}(1)]^2 \approx \mathbf{z}(1)^2 + 2\mathbf{z}(1)\Delta\mathbf{z}(1) \\ y^{o2} = [\mathbf{z}(2) + \Delta\mathbf{z}(2)]^2 \approx \mathbf{z}(2)^2 + 2\mathbf{z}(2)\Delta\mathbf{z}(2) \\ z^{o2} = [\mathbf{z}(3) + \Delta\mathbf{z}(3)]^2 \approx \mathbf{z}(3)^2 + 2\mathbf{z}(3)\Delta\mathbf{z}(3) \\ x^{o2} + y^{o2} + z^{o2} = \mathbf{z}(4) + \Delta\mathbf{z}(4) \end{cases} \quad (21)$$

In equation (21), $\mathbf{z}(k)$, $\Delta\mathbf{z}(k)$ represent the k -th element of the vector \mathbf{Z} , $\Delta\mathbf{z}$, $k = 1, 2, 3, 4$. Express equation (21) as a linear matrix form:

$$\mathbf{G}\boldsymbol{\mu}^o = \mathbf{h} + \boldsymbol{\eta} \quad (22)$$

In equation $\mathbf{h} = [z(1)^2 \ z(2)^2 \ z(3)^2 \ z(4)^2]^T$, $\boldsymbol{\mu}^o = [x^{o2} \ y^{o2} \ z^{o2}]^T$, $\boldsymbol{\eta} = \mathbf{L}\Delta\mathbf{z}$

$$\mathbf{L} = \text{diag}\{2z(1) \ 2z(2) \ 2z(3) \ 1\}$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}^T$$

According to the principle of linear least squares, the unbiased estimate of the vector $\boldsymbol{\mu}^o$ is:

$$\boldsymbol{\mu}^o = (\mathbf{G}^{-1} \boldsymbol{\Sigma}_\eta^{-1} \mathbf{G})^{-1} \mathbf{G} \boldsymbol{\Sigma}_\eta^{-1} \mathbf{h} \quad (23)$$

The weight matrix $\boldsymbol{\Sigma}_\eta$ value of the dimension 4×4 in equation (23) is:

$$\boldsymbol{\Sigma}_\eta = \boldsymbol{\Sigma}(\boldsymbol{\eta}^T \boldsymbol{\eta}) = \mathbf{L}^T \text{cov}(\Delta\mathbf{z}) \mathbf{L} = \mathbf{L}^T (\mathbf{A}^T \boldsymbol{\Sigma}_\gamma^{-1} \mathbf{A})^{-1} \mathbf{L} \quad (24)$$

Due to then the exact estimated value of the coordinates of the location node being located is

$$\mathbf{x} = \text{sign}\{\text{diag}\{\mathbf{z}(1:3)\}\} \sqrt{\boldsymbol{\mu}^o} \quad (25)$$

The calculation process represented by the equation (25) takes into account the mutual constraint relationship between the elements in the parameter \mathbf{z} , and obtains accurate positioning results. This calculation method is called the constrained linear least squares (CLLS) method of the asynchronous sensor network positioning problem of the RSS and TOA hybrid measurement methods.

Model of the Cramer-Rao Lower Bound (CRLB)

The CRLB value provides the lower bound of the error variance for the unbiased estimation of the model to be estimated, and the unknown parameter of the model is assumed to be $\boldsymbol{\rho} = [\mathbf{x} \ \omega \ \theta]$. Assuming that the estimated error variance of unknown parameters $\boldsymbol{\rho}$ is $\text{cov}(\boldsymbol{\rho})$. Then according to Cramer-Rao lower bound theory, there is a relationship $\text{cov}(\boldsymbol{\rho}) \geq \mathbf{F}^{-1}$, among them \mathbf{F} is the representation of the FIM (Fisher Information Matrix) of parameter to be evaluated $\boldsymbol{\rho}$, the matrix \mathbf{F} expressed as:

$$\mathbf{F} = - \frac{\partial^2 \ln P(\boldsymbol{\rho} | \mathbf{p})}{\partial \boldsymbol{\rho}^T \partial \boldsymbol{\rho}} \quad (26)$$

The model is solved by RSS, TOA measures parameters, and maximizing the matrix \mathbf{F} . The probability density function $\ln P(\boldsymbol{\rho} | \mathbf{p})$ can be expressed as:

$$P(\boldsymbol{\rho} | \mathbf{p}) = \prod_{i=1}^N \frac{1}{\sqrt{2\pi}\delta_{i,m}} \exp\left(-\frac{r_{i,m}^2}{2\delta_{i,m}^2}\right) \frac{1}{\sqrt{2\pi}\delta_{i,n}} \exp\left(-\frac{r_{i,n}^2}{2\delta_{i,n}^2}\right) \frac{1}{\sqrt{2\pi}\delta_{i,\varepsilon}} \exp\left(-\frac{r_{i,\varepsilon}^2}{2\delta_{i,\varepsilon}^2}\right) \quad (27)$$

Take the logarithm of both sides of equation (27) and define the vector \mathbf{r}_m and \mathbf{r}_n , \mathbf{r}_ε , $\mathbf{r}_m = [r_{1,m} \ r_{2,m} \ \dots \ r_{N,m}]$, $\mathbf{r}_n = [r_{1,n} \ r_{2,n} \ \dots \ r_{N,n}]$, $\mathbf{r}_\varepsilon = [r_{1,\varepsilon} \ r_{2,\varepsilon} \ \dots \ r_{N,\varepsilon}]$ with the following expression:

$$\ln P(\boldsymbol{\rho} | \mathbf{p}) = e - (\mathbf{r}_m^T \boldsymbol{\Sigma}_m^{-1} \mathbf{r}_m + \mathbf{r}_n^T \boldsymbol{\Sigma}_n^{-1} \mathbf{r}_n + \mathbf{r}_\varepsilon^T \boldsymbol{\Sigma}_\varepsilon^{-1} \mathbf{r}_\varepsilon) \quad (28)$$

In equation (28), $\boldsymbol{\Sigma}_m = \text{diag}\{\delta_{i,m}^2\}$, $\boldsymbol{\Sigma}_n = \text{diag}\{\delta_{i,n}^2\}$, $\boldsymbol{\Sigma}_\varepsilon = \text{diag}\{\delta_{i,\varepsilon}^2\}$, $i = 1, 2, \dots, N$, e is the constant part. Substituting equation (28) into equation (26) and the matrix \mathbf{F} can be expressed as:

$$\mathbf{F} = \left(\frac{\partial \mathbf{r}_m}{\partial \boldsymbol{\rho}}\right)^T \boldsymbol{\Sigma}_m^{-1} \frac{\partial \mathbf{r}_m}{\partial \boldsymbol{\rho}} + \left(\frac{\partial \mathbf{r}_n}{\partial \boldsymbol{\rho}}\right)^T \boldsymbol{\Sigma}_n^{-1} \frac{\partial \mathbf{r}_n}{\partial \boldsymbol{\rho}} + \left(\frac{\partial \mathbf{r}_\varepsilon}{\partial \boldsymbol{\rho}}\right)^T \boldsymbol{\Sigma}_\varepsilon^{-1} \frac{\partial \mathbf{r}_\varepsilon}{\partial \boldsymbol{\rho}} \quad (29)$$

In equation (29), $\frac{\partial \mathbf{r}_m}{\partial \boldsymbol{\rho}}$, $\frac{\partial \mathbf{r}_n}{\partial \boldsymbol{\rho}}$, $\frac{\partial \mathbf{r}_\varepsilon}{\partial \boldsymbol{\rho}}$ are respectively the differential of each vector to the unknown parameter $\boldsymbol{\rho}$, which can be written as follows:

$$\begin{cases} \frac{\partial \mathbf{r}_m}{\partial \boldsymbol{\rho}} = \begin{bmatrix} \mathbf{r}_{1,m}^\rho & \mathbf{r}_{2,m}^\rho & \dots & \mathbf{r}_{N,m}^\rho \end{bmatrix} \\ \frac{\partial \mathbf{r}_n}{\partial \boldsymbol{\rho}} = \begin{bmatrix} \mathbf{r}_{1,n}^\rho & \mathbf{r}_{2,n}^\rho & \dots & \mathbf{r}_{N,n}^\rho \end{bmatrix} \\ \frac{\partial \mathbf{r}_\varepsilon}{\partial \boldsymbol{\rho}} = \begin{bmatrix} \mathbf{r}_{1,\varepsilon}^\rho & \mathbf{r}_{2,\varepsilon}^\rho & \dots & \mathbf{r}_{N,\varepsilon}^\rho \end{bmatrix} \end{cases} \quad (30)$$

In equation (30), $\mathbf{r}_{i,m}^\rho$, $\mathbf{r}_{i,n}^\rho$, $\mathbf{r}_{i,\varepsilon}^\rho$ further expansion of the solution is expressed:

$$\begin{cases} \mathbf{r}_{i,m}^\rho = \frac{\partial r_{i,m}}{\partial \boldsymbol{\rho}} = \begin{bmatrix} \mathbf{x}_i - \mathbf{x} & T_{i,i} - \theta & 1 \\ cd_i & \omega^2 & \omega \end{bmatrix} \\ \mathbf{r}_{i,n}^\rho = \frac{\partial r_{i,n}}{\partial \boldsymbol{\rho}} = \begin{bmatrix} \mathbf{x}_i - \mathbf{x} & \theta - R_{2,i} & -1 \\ cd_i & \omega^2 & \omega \end{bmatrix} \\ \mathbf{r}_{i,\varepsilon}^\rho = \frac{\partial r_{i,\varepsilon}}{\partial \boldsymbol{\rho}} = \begin{bmatrix} 4.34\beta(\mathbf{x} - \mathbf{x}_i)^T & 0 & 0 \\ d_i^2 & & \end{bmatrix} \end{cases} \quad (31)$$

Then according to the CRLB unbiased estimate lower bound theory has

$$\text{CRLB}([\boldsymbol{\rho}]_p) = [\mathbf{F}^{-1}]_{p,p} \quad (32)$$

In the equation (32) $p = 1, 2, \dots, 5$, $[\mathbf{F}^{-1}]_{p,p}$ represents the p -th row and the p -th column element value of the inverse matrix of \mathbf{F} ; $\text{CRLB}([\mathbf{p}]_p)$ indicates the lower bound of the CRLB unbiased to estimate of the P row element of unknown parameters \mathbf{p}

Simulation Analysis

In this section, numerical results will be provided to verify and compare the feasibility of the proposed non-synchronous sensor network positioning method based on RSS and TOA hybrid measurements. According to the calculation method of the above design, the hybrid positioning algorithm is simulated by MATLAB software, and the simulation results are more intuitively displayed and analyzed. Assume that the clock of the set beacon nodes are synchronized, that are $\omega_i = 1$, $\theta_i = 0$, $i = 1, 2, \dots, N$ while the unknown node has clock drift and deviation. The time measurement noise variance $\delta_{i,m}$ with $\delta_{i,n}$, RSS noise variance $\delta_{i,\epsilon}$, signal emission intensity $p_0 = -45\text{dB}$, path attenuation index

$\beta = 4$ are set in advance between the unknown nodes and beacon nodes i . The accuracy of the positioning error of the designed algorithm is based on the Root Mean Square Error (RMSE) analysis. The simulation runs 1000 times, and the average value of the results is used for error analysis.

The influence of the number of beacon nodes on the positioning error

Assuming that the simulation environment is in a 3D region of $100\text{ m} \times 100\text{ m} \times 100\text{ m}$, the coordinates of the unknown node is set to $(50, 50, 50)$ in advance. Because the quantity of beacon nodes is too few lead to the positioning accuracy is too low, so the number of beacon nodes is set as $N = 4, 5, \dots, 8$. Table 1 compares the Unconstrained Linear Least Squares (ULLS) and constrained least squares (CLLS) positioning accuracy of the three different methods of RSS, TOA, RSS/TOA, and the CRLB value is compared as a system benchmark.

Table 1: Comparison of the influence of the number of beacon nodes on the positioning error

		4	5	6	7	8
ULLS	RSS	0.873	0.628	0.531	0.456	0.377
	TOA	0.718	0.573	0.359	0.272	0.185
	RSS/TOA	0.141	0.113	0.109	0.089	0.075
CLLS	RSS	0.591	0.504	0.438	0.356	0.271
	TOA	0.405	0.311	0.237	0.188	0.139
	RSS/TOA	0.063	0.039	0.033	0.028	0.025
CRLB	RSS	0.512	0.425	0.351	0.288	0.204
	TOA	0.323	0.255	0.189	0.132	0.093
	RSS/TOA	0.047	0.024	0.021	0.019	0.018

Table 1 lists 1000 simulations under the premise that the time measurement noise variance $\delta_{i,m}$, $\delta_{i,n}$ and RSS noise variance $\delta_{i,\epsilon}$ are all 0.2, and compares the positioning errors of the ULLS and CLLS calculation methods mentioned above in the paper and compared them with CRLB. It can be seen from Table 1 that with the increase of the number of beacon nodes, the measurement connection relationship between nodes increases, and both algorithms and CRLB values decrease accordingly. For example, when the number of beacon nodes is 5, the positioning error of the single RSS in the ULLS method is 0.628 m, and that in the CLLS method is 0.504 m; the positioning error of the single TOA in the ULLS method is 0.573 m, and that in the CLLS method is 0.311 m; The positioning error of the hybrid measured RSS and TOA in the ULLS method is 0.425 m, and that in the CLLS method is 0.255 m. This shows that in the case of the same number of beacon nodes, whether in the unconstrained linear least squares or constrained least squares method, the error of the hybrid measurement method is smaller than the single measurement method (RSS or TOA), which indicates the hybrid measurement method is more accurate. It can also be seen from Table 1 that the proposed hybrid method yield results very close to CRLB, which confirms the feasibility and accuracy of the hybrid measurement method, proposed in this paper.

Influence of noise on positioning error

In the linear estimation method proposed in this paper, each noise of time measurement and signal strength measurement has independence, which leads to a decrease in the accuracy of the positioning method. In order to test the estimation error of the ULLS and CLLS calculation methods proposed in this paper, the positioning accuracy of the algorithm is compared with CRLB. And the results are compared with the SDP method introduced in literature [16]. In order to make the simulation results more representative, the number of beacon nodes is selected to be 5, and keeping the time measurement noise variance $\delta_{i,m}$ with $\delta_{i,n}$ both are 0.2 ns, while adjusting RSS measurement noise $\delta_{i,\epsilon}$ varies from 0.2 dB to 2 dB. Figure 2 depicts the relation of RMSE positioning error with RSS measurement noise under different algorithms. As shown in figure 2, the value of the RSS measurement noise has a great influence on the positioning accuracy of the algorithm. With the increases of the RSS measurement noise, the positioning error also increases. When the RSS measurement noise is equal to 0.2 dB, the positioning error RMSE of the designed ULLS method is 0.131 m, and the positioning error of the CLLS method is only 0.039 m. The constrained CLLS method is more accurate than the ULLS method, and its positioning result is closer to the CRLB

value. Compared with the SDP method, the positioning error of the ULLS method is larger, and the positioning error of the CLLS method is smaller than that of the SDP method. When the RSS measurement noise is equal to 0.2 dB, the positioning error of SDP method is 0.075 m, which is between the positioning error of ULLS and CLLS method.

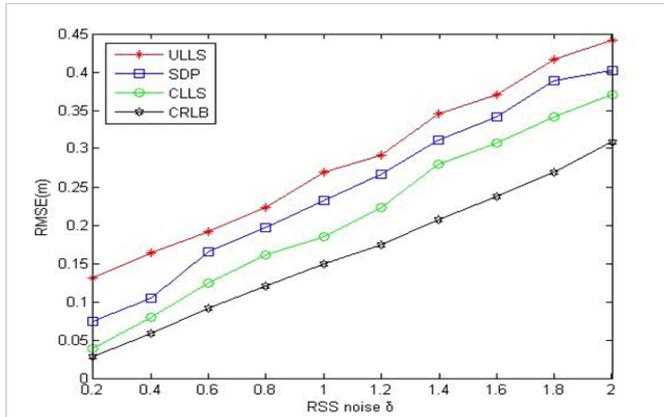


Figure 2: The effect of RSS measurement noise on positioning error

Similarly, keep adjusting RSS measurement noise $\delta_{i,n}$ to 0.2 dB, the time measurement noise variance $\delta_{i,m}$ with $\delta_{i,n}$ both vary from 0.2 ns to 2 ns at the same time. It can be seen from figure 3 that the RMSE positioning error under different algorithms increases as the TOA measurement noise increases. Compared with the influence of RSS measurement noise on positioning error, TOA measurement noise has less influence on positioning error. When the TOA measurement noise is equal to 0.2 ns, the positioning error RMSE of the designed ULLS method is 0.107 m, the positioning error of the SDP method is 0.058, and the positioning error of the CLLS method is only 0.031 m. The constrained CLLS method is closer to the CRLB than the ULLS method. The same analysis results as in Fig. 2 show that the positioning error using the constrained CLLS method is smaller than that of the ULLS and SDP methods.

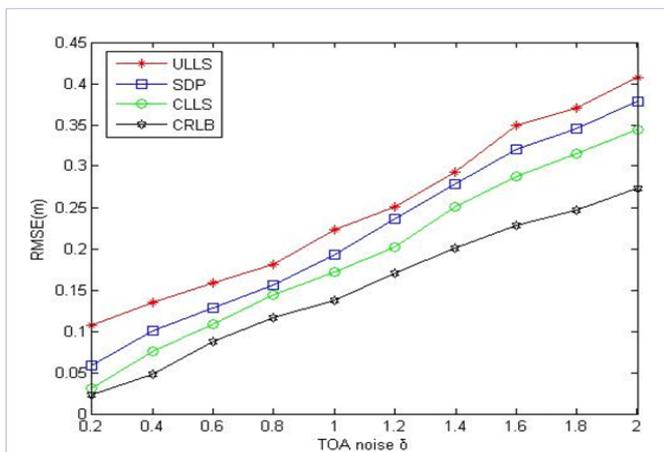


Figure 3: The effect of TOA measurement noise on positioning error

Influence of path attenuation index on positioning error

In order to make the simulation results more representative, this paper compares the influence of path attenuation index on positioning error and sets the RSS measurement noise $\delta_{i,\epsilon}$ to 0.2 dB, the time measurement noise variance $\delta_{i,m}$ and $\delta_{i,n}$ are set to 0.2 ns, the path attenuation index β value is obtained by experimental analysis in advance, and its value varies with the change of the propagation medium, generally between the typical value of 2 and 5, and adjusts β value changes within the interval.

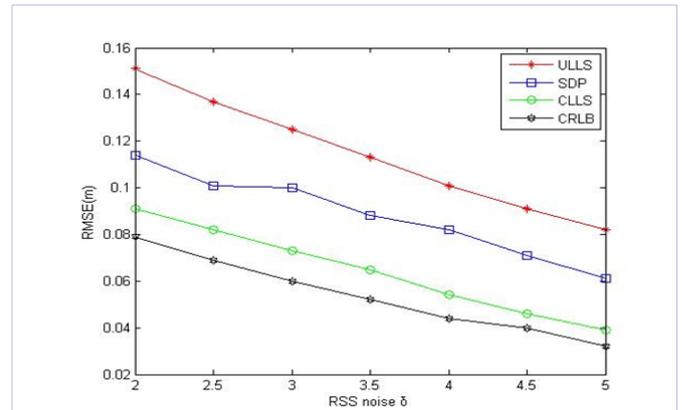


Figure 4: Influence of path attenuation index on positioning error

Figure 4 plots the variation of the RMSE positioning error with the path attenuation index under different algorithms. It can be seen from the figure that the positioning error gradually decreases with the path attenuation index β value increases. When the path attenuation index β value is equal to 2, the positioning error RMSE of the designed ULLS method is 0.151 m, the positioning error of the SDP method is 0.114m, the positioning error of the constrained CLLS method is 0.091 m, and the CRLB value is 0.079 m; when the path attenuation index β value is equal to 5, the positioning error RMSE of the designed ULLS method is 0.082 m, the positioning error of the SDP method is 0.061m, the positioning error of the constrained CLLS method is 0.039 m, and the CRLB value is 0.032 m.

Time synchronization parameter estimation error

The model designed in this paper assumes that the clock parameters of the beacon nodes, including the drift rate, deviation amount are calibrated, while the unknown node clock synchronization parameter is unknown parameter. The time synchronization parameters of the unknown node including the drift rate, the deviation amount, and the position coordinates of the unknown node are simultaneously estimated. Because the noise error change of RSS in the hybrid algorithm has little effect on the clock drift rate and deviation amount, the fixed RSS noise error value is selected to be 0.2 dB, which makes the TOA noise error change from 0.2 to 2 ns. The clock drift rate and deviation amount of the unknown node are selected to analyze the estimation error. The results are shown in Figure 5 and Figure 6.

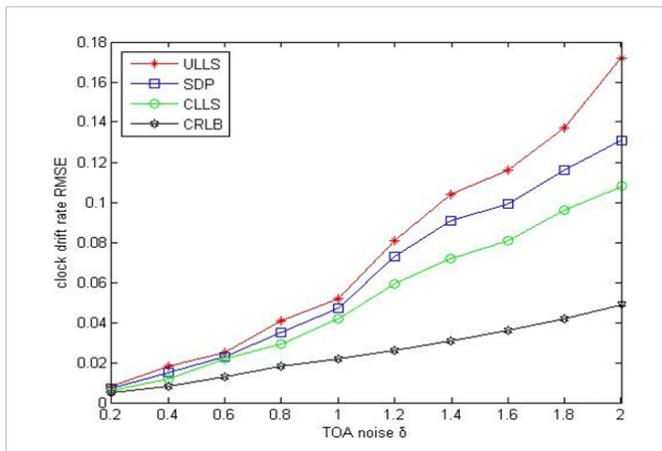


Figure 5: Clock drift rate estimation error

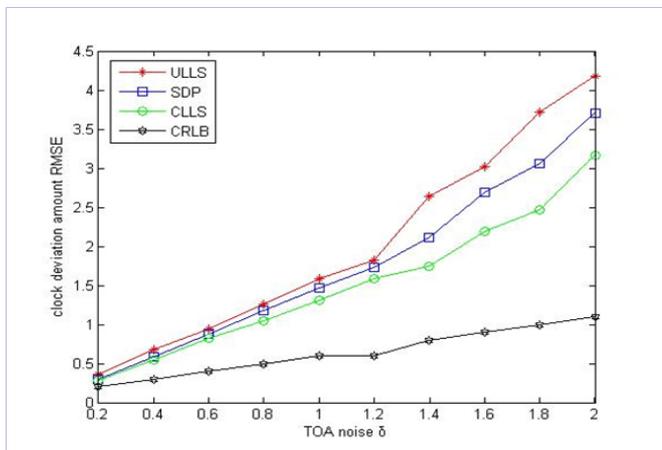


Figure 6: Clock deviation amount t estimation error

Observing the change curve of the clock drift rate and deviation amount of Figure 5 and Figure 6 with noise $\delta_{i,m}$ and $\delta_{i,n}$, it is not difficult to find that the variation law of the algorithm is similar to the result reflected in Figure 3. The general trend is that the clock drift rate increases with the increase of noise, which is an upward trend. As when $\delta_{i,m}$ and $\delta_{i,n}$ are 0.2 ns, the clock drift rate RMSE of the ULLS is 0.008, the clock drift rate RMSE of the SDP is 0.007, while the clock drift rate RMSE of the CLLS is 0.006; when $\delta_{i,m}$ and $\delta_{i,n}$ are 2 ns, the clock drift rate RMSE of the ULLS increases to 0.172, the clock drift rate RMSE of the SDP increases to 0.131, while the CLLS clock drift rate RMSE increases to 0.125.

Similarly, the clock deviation quantity estimation error is similar to the above-mentioned result, and the overall trend is also increasing with the increase of noise. As when $\delta_{i,m}$ and $\delta_{i,n}$ are 0.2 ns, the clock deviation quantity RMSE of the ULLS is 0.36, the clock deviation quantity RMSE of the SDP is 0.29, while the clock deviation quantity RMSE of the CLLS is 0.28; and when $\delta_{i,m}$ and $\delta_{i,n}$ are 2 ns, the clock deviation quantity RMSE of ULLS increases to 4.18, the clock deviation quantity RMSE of SDP increases to 3.71, while the CLLS clock deviation quantity RMSE increases to 3.17.

Conclusion

This paper proposes a 3D asynchronous sensor network positioning method based on hybrid measurement of RSS and TOA. Using the clock drift and deviation TOA ranging model, the time synchronization parameters and the node position coordinates are simultaneously estimated, which realizes both time synchronization and node position coordinate estimation. Converting the nonlinear optimization model of RSS and TOA hybrid measurement into linear equation, obtains the Unconstrained Linear Least Squares linear method (ULLS) of the unknown node position coordinates, and optimizes the positioning result of the ULLS. And convert it to a more precise Constrained Least Squares (CLLS) result. Compared with the SDP method introduced in the literature, the proposed ULLS and CLLS linear estimation method calculation does not depend on the choice of the initial solution, with low complexity and fast operation speed. The simulation results show that the performance of the proposed hybrid positioning algorithm is superior to the traditional single RSS or TOA method. Compared with the SDP method, the error of the constrained CLLS method is smaller, which is closer to the CRLB lower bound value of the positioning result. However, the ULLS and CLLS obtained by the hybrid positioning method proposed in this paper have certain limitations, and the estimation error has not reached the CRLB, which needs further optimization.

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