Effects of Maxwellian ions on Dust-acoustic Solitary Waves in Adiabatic Degenerate Plasmas and It's Instability

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Abstract

A rigorous theoretical investigation has been made on dust acoustic solitary waves in unmagnetized and magnetized degenerate plasmas with adiabatic pressure. Korteweg-de Vries (K-dV) equation have been derived for unmagnetized case as well as its solution. For the case of magnetized plasmas, Zakharov-Kuznetsov (ZK) equation, and its solution have been derived and studied its instability criterion and growth rate. It has been found that, the parametric regimes effects not only modify the basic properties of dust acoustic solitary waves and its instability criterion but also introduce some important new features.

Keywords: Maxwellian ions; Dust-acoustic Solitary Waves; Degenerate Plasmas; K-dV equation; ZK equation; Instability;

Introduction

The propagation of dust acoustic waves (DASW's) waves plays an important role in understanding the different behaviour of dust components or different waves phenomena in dusty plasmas which are omnipresent in laboratory, space and astrophysical plasma environments, such as cometary tails, planetary ring, interstellar medium etc. The dust grain changes the nature of the system including the creation of the new modes. In the recent years, many important and precious, theoretical and experimental investigation has been made on solitary waves. DASW's are one of the most significant non-linear affair in plasma method. Different properties such as amplitude, width etc. of the solitary waves can be modified by using different perturbation method. Bliokh and Yaroshenko studied the electrostatic waves in dusty plasmas and applied their results in interpreting spoke-like structures in Saturn's rings (revealed by the voyager space mission)[1, 2]. Rao et.al, were the first to predict theoretically existence of extremely lower phase velocity DASW's in unmagnetized dusty plasmas whose constituent are inertial charged dust grains and Boltzmann distributed ions and electrons [3]. Roychoudhury and Mukherjee considered a two-component unmagnetized

dusty plasma consisting of a negatively charged adiabatic dust fluid and an inertia-less isothermal ion fluid and investigated the effects of dust fluid temperature on large amplitude solitary waves by the pseudo-potential approach [4]. Sayed and Mamun assumed a dusty plasma containing the adiabatic dust fluid and non-adiabatic (isothermal) inertia-less electron and ion fluid and studied the effect of the dust fluid temperature on the DASW's by the reductive perturbation method [5, 6]. Mendoza-Briceño et al, assumed a two-component dusty plasma containing the adiabatic dust fluid and non-adiabatic ions following the nonthermal distribution of Cairns et al and studied the effect of the dust fluid temperature on the DA solitary waves by the pseudo-potential approach [7-9].

The equation of state for degenerate electrons in such interstellar compact objects are mathematically explained by Chandrasekhar for two limits, namely non-relativistic and ultra-relativistic limits [10,11]. The degenerate electron equation of state of Chandrasekhar is P_e is proportional to $N_e^{5/3}$ for the non-relativistic limit and P_e is proportional to $N_e^{4/3}$ for the ultra-relativistic limit where P_e is the degenerate electron pressure and N_e is the degenerate electron number density. Mamun and Shukla considered an unmagnetized degenerate plasma without and with negatively charged stationary dust [12, 13]. Mamun et al, studied the ion acoustic Solitary Waves (SW's) in the presence of an external magnetic field for ultra-relativistic degenerate electron-ion plasmas using the reductive perturbation technique [14]. Using the same technique Haider et al, also studied the obliquely propagating solitary structure with the presence of external magnetic field [15,16]. But the presence of heavy ions, which can be arbitrary charged, were not considered in the work of Mamun et al and Haider et al [14,15].

In the present work we have studied the nonlinear propagation of DA SW's in unmagnetized and magnetized degenerate plasmas with adiabatic pressure. We have studied both ultra-relativistic and non-relativistic case simultaneously using the generalized equation proposed by Haider [17]. To do this we have derived Korteweg-de Vries (K-dV) and Zakharov-Kuznetsov (ZK) equation by reductive perturbation method and find out the solution of it [6]. We have studied the instability criterion as well as it's growth rate.

Plasma Model

We have studied the nonlinear propagation of DASW's in unmagnetized and magnetized degenerate plasmas for adiabatic situation containing,

- (i) inertial positive mobile dust component
- (ii) inertialess degenerate electrons
- (iii) inertialess degenerate positrons
- (iv) stationary negative dust component
- (v) inertial Maxwellian ions

At equilibrium, we have,

$$n_{pd0} + n_{i0} + n_{p0} = n_{nd0} + n_{e0} \quad (1)$$

where, n_{pd0} , n_{i0} , n_{p0} , n_{nd0} , n_{e0} are the positive dust, ion, positron, negative dust and electron number densities at equilibrium respectively.

Solitary Waves in Unmagnetized Plasmas

Basic Equations

The dynamics of such DASW's in one dimensional form and is given by the followings equations,

$$\frac{\partial n_{pd}}{\partial t} + \frac{\partial}{\partial x}(n_{pd}u_{pd}) = 0 \quad (2)$$

$$\frac{\partial u_{pd}}{\partial t} + (u_{pd} \frac{\partial}{\partial x})u_{pd} = -\frac{\partial \phi}{\partial x} - (\frac{\sigma_d}{n_{pd}}) \frac{\partial}{\partial x} p_{pd} \quad (3)$$

$$\frac{\partial p_{pd}}{\partial t} + (u_{pd} \frac{\partial}{\partial x})p_{pd} + \alpha p_{pd} \frac{\partial}{\partial x} u_{pd} = 0 \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (\mu_e n_e - \mu_p n_p - n_{pd} - \mu_i n_i + \mu_{nd} n_{nd}) \quad (5)$$

where, n_{pd} is the positive dust number density normalized by its equilibrium value n_{pd0} , n_{nd} is the negative dust number density normalized by its equilibrium value n_{nd0} , n_p is the positron number density normalized by its equilibrium value n_{p0} , n_i is the ion number density normalized by its equilibrium value n_{i0} , n_e is the electron number density normalized by its equilibrium value n_{e0} , u_{pd} is the dust fluid speed normalized by $C_{pd} = (m_{nd}c^2 / m_{pd})^{1/2}$, m_{pd} is the rest mass of positive dust, m_{nd} is the mass of negative dust and C being the speed of light. ϕ is the DA electrostatic wave potential normalized by $(m_{pd}C_{pd}^2 / z_d e)$, with z_d is the number of positive charge residing on the positive charged dust and e being the magnitude of unit charge. α is the adiabatic index. σ_d is the ratio of the positive dust temperature to negative dust temperature. $\mu_e = n_{e0}/n_{pd0}$, $\mu_p = n_{p0}/n_{pd0}$, $\mu_i = n_{i0}/n_{pd0}$ and $\mu_{nd} = n_{nd0} / n_{pd0}$

Here the space variables are normalized by Debye radius

$$\lambda_d = (m_{pd}c^2 / 4\pi n_{d0}z_d^2 e^2)^{1/2} \text{ and time variable } (t) \text{ is normalized by } \omega_{pd}^{-1} = (m_{pd} / 4\pi n_{pd0}z_d^2 e^2)^{1/2}.$$

Non-thermal distributed ions can be represented by,

$$n_i = (1 - \phi + \frac{1}{2}\phi^2) \quad (6)$$

According to Haider the value of n_e and n_p can be express respectively as [18, 19],

$$n_e = (1 + \frac{\gamma-1}{\beta}\phi)^{\frac{1}{\gamma-1}} \quad (7)$$

$$\text{and } n_p = (1 - \frac{\gamma-1}{\beta}\phi)^{\frac{1}{\gamma-1}} \quad (8)$$

where, $\beta = (\frac{K}{E})n_0^{(\gamma-1)}$ where, $K = \frac{1}{18(\gamma-1)}(\frac{3\Delta^3}{\pi})^{(\gamma-1)}E$ with $\Lambda(= \frac{h}{m_{pd}c})$ is the Compton wavelength normalized by Debye radius (λ_d) , $E = m_{pd}c^2$ and n_0 be the number density of plasma particle. $\gamma = 4/3$ for ultra-relativistic case and $\gamma = 5/3$ for non-relativistic case.

K-dV Equation

We now follow the reductive perturbation method and construct a weakly nonlinear theory for the DA waves with small but finite amplitude, which leads to a scaling of the independent variables through the stretched coordinates as [6, 18, 19],

$$Z = \varepsilon^{1/2}(x - v_p t) \quad (9)$$

$$\tau = \varepsilon^{3/2}t \quad (10)$$

where, ε is a smallness parameter measuring the weakness of the dispersion, v_p is the nonlinear wave phase velocity. We can expand the perturbed quantities n_{pd} , u_{pd} , p_{pd} and ϕ about their equilibrium values in powers of ε ,

$$n_{pd} = 1 + \varepsilon n_{pd}^{(1)} + \varepsilon^2 n_{pd}^{(2)} + \dots \quad (11)$$

$$u_{pd} = 0 + \varepsilon u_{pd}^{(1)} + \varepsilon^2 u_{pd}^{(2)} + \dots \quad (12)$$

$$\phi = 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots \quad (13)$$

$$p_{pd} = 1 + \varepsilon p_{pd}^{(1)} + \varepsilon^2 p_{pd}^{(2)} + \dots \quad (14)$$

Using the stretched coordinates and equations (11)-(14) in equations (2)-(5); and equating the coefficients of $\varepsilon^{3/2}$ from the continuity and momentum equation and coefficients of ε from Poisson's equation and rearranging the parameters one can obtain the linear dispersion relation for the DASW's

$$v_p = \sqrt{\frac{1}{\left(\frac{1}{\beta}\mu_e + \frac{1}{\beta}\mu_p + \mu_i\right)}} + \alpha\sigma_d \quad (15)$$

It can be seen from equation (15) that the presence of the Maxwellian ions significantly modifies the linear dispersion relation.

Equating the next higher order co-efficient of ε from above equations and using the parameters we can finally obtain a K-dV equation describing the nonlinear propagation of the DASW's in the dusty plasma

Figure 1: $A=0$ graph, showing the variation of β with μ_e and μ_p for the values of $\alpha = 0.5$, $\mu_i = 0.4$ and $\gamma = 5/3$

Figure 2: $A=0$ graph, showing the variation of β with μ_e and μ_p for the values of $\alpha = 0.5$, $\mu_i = 0.4$ and $\gamma = 4/3$

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial Z} + B \frac{\partial^3 \phi^{(1)}}{\partial Z^3} = 0 \quad (16)$$

where,

$$A = \left[\mu_i - (\mu_e - \mu_p) \left(\frac{2-\gamma}{\beta^2} \right) \right] \frac{(v_p^2 - \alpha\sigma_d)^2}{2v_p} \quad (17)$$

$$B = \frac{(v_p^2 - \alpha\sigma_d)^2}{2v_p} \quad (18)$$

Solution of K-dV Equation

The stationary solution of this K-dV equation can be obtained by transforming the independent variables ξ and τ to $\xi = Z - u_0 \tau$, $\tau = t$, where u_0 is a constant solitary wave velocity. For simplicity, we have write $\phi^{(1)}$ as ϕ . Now using the appropriate boundary conditions for localized disturbances, viz.

$$\phi^{(1)} \rightarrow 0, (d\phi^{(1)} / d\xi) \rightarrow 0, (d^2\phi^{(1)} / d\xi^2) \rightarrow 0$$

at $\xi \rightarrow \pm\infty$. Thus, one can express the stationary solution of this K-dV equation as

$$\phi = \phi_m \text{sech}^2[\xi/\Delta] \quad (19)$$

where $\phi_m = (3u_0 / A)$ is the amplitude and $\Delta = \sqrt{\frac{4B}{u_0}}$ is the width of the solitary waves. The value of A and B are shown in (17) and (18) respectively.

Figure 3: Variation of ϕ with ξ for the values of $\alpha = 0.5$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1.2$, $u_0 = 0.1$, $\sigma_d = 1$, $\beta = 1$ $\gamma = 5/3$ (solid curve) and $\gamma = 4/3$ (dotted curve).

Figure 4: Variation of ϕ with ξ for the values of $\alpha = 0.5$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1.2$, $u_0 = 0.1$, $\sigma_d = 1$, $\beta = 0.8$ (Black), $\beta = 0.9$ (Blue) and $\beta = 1$ (Red). Here dotted curve represented ultra-relativistic ($\gamma = 4/3$) case and solid curve represented non-relativistic ($\gamma = 5/3$) case.

Solitary Waves in Magnetized Plasmas

Basic Equations

We have been considered that, there an external static magnetic field present B_0 acting along the z-direction $B_0 = \hat{k}B_0$)

where $\hat{k}$ is the unit vector along the z-direction, so that the ions and dusts are moving along the magnetic field direction.

The dynamics of such DASW's in three dimensional form and is given by the followings equations,

$$\frac{\partial n_{pd}}{\partial t} + \nabla \cdot (n_{pd} u_{pd}) = 0 \quad (20)$$

$$\frac{\partial u_{pd}}{\partial t} + (u_{pd} \cdot \nabla) u_{pd} = -\nabla \varphi + \omega_{cd} (u_{pd} \times \hat{k}) - \left(\frac{\sigma_d}{n_{pd}} \right) \nabla p_{pd} \quad (21)$$

$$\frac{\partial p_{pd}}{\partial t} + (u_{pd} \cdot \nabla) p_{pd} + \alpha p_{pd} \nabla u_{pd} = 0 \quad (22)$$

$$\nabla^2 \varphi = (\mu_e n_e - \mu_p n_p - n_{pd} - \mu_i n_i + \mu_{nd} n_{nd}) \quad (23)$$

Here, ω_{cd} is the positive dust cyclotron frequency ($z_d e B_0 / m_{pd} c$) normalized by

$$\omega_{pd} = \left(\frac{4\pi n_{pd0} z_d^2 e^2}{m_{pd}} \right)^{\frac{1}{2}}.$$

Figure 5: Variation of φ with ξ for the values of $\beta=1$, $\mu_p=1$, $\mu_i=0.4$, $\mu_e=1.2$, $u_0=0.1$, $\sigma_d=1$, $\alpha=0.3$ (Black), $\alpha=0.4$ (Red) and $\alpha=0.5$ (Blue). Here dotted curve represented ultra-relativistic ($\gamma=4/3$) case and solid curve represented non-relativistic ($\gamma=5/3$) case.

Figure 6: Variation of φ with ξ for the values of $\beta=1$, $\alpha=0.5$, $\mu_p=1$, $\mu_i=0.4$, $u_0=0.1$, $\mu_e=1.2$, $\sigma_d=0.6$ (Black), $\sigma_d=0.8$ (Red) and $\sigma_d=1$ (Blue). Here dotted curve represented ultra-relativistic ($\gamma=4/3$) case and solid curve represented non-relativistic ($\gamma=5/3$) case.

Zakharov-Kuznetsov Equation

To derive the Zakharov-Kuznetsov equation, which is known as 'K-dV equation in three dimensions', we first introduce the following stretched coordinates

$$X = \varepsilon^{1/2} x \quad (24)$$

$$Y = \varepsilon^{1/2} y \quad (25)$$

$$Z = \varepsilon^{1/2} (z - v_p t) \quad (26)$$

$$\tau = \varepsilon^{3/2} t \quad (27)$$

We can expand the perturbed quantities

n_{pd} , u_{pdx} , u_{pdy} , u_{pdz} , p_{pd} and φ about their equilibrium values in powers of ε

$$n_{pd} = 1 + \varepsilon n_{pd}^{(1)} + \varepsilon^2 n_{pd}^{(2)} + \dots \quad (28)$$

$$u_{pdx} = 0 + \varepsilon^{3/2} u_{pdx}^{(1)} + \varepsilon^2 u_{pdx}^{(2)} + \dots \quad (29)$$

$$u_{pdy} = 0 + \varepsilon^{3/2} u_{pdy}^{(1)} + \varepsilon^2 u_{pdy}^{(2)} + \dots \quad (30)$$

$$u_{pdz} = 0 + \varepsilon u_{pdz}^{(1)} + \varepsilon^2 u_{pdz}^{(2)} + \dots \quad (31)$$

$$\varphi = 0 + \varepsilon \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \dots \quad (32)$$

$$p_{pd} = 1 + \varepsilon p_{pd}^{(1)} + \varepsilon^2 p_{pd}^{(2)} + \dots \quad (33)$$

Figure 7: Variation of φ with ξ for the values of $\beta=1$, $\alpha=0.5$, $\mu_i=0.4$, $u_0=0.1$, $\mu_e=1.2$, $\sigma_d=1$, $\mu_p=0.8$ (Blue), $\mu_p=0.9$ (Black) and $\mu_p=1$ (Red). Here dotted curve represented ultra-relativistic ($\gamma=4/3$) case and solid curve represented non-relativistic ($\gamma=5/3$) case.

Next, substituting eqs. (24)-(33) into eqs. (20)-(23) and obtain the lowest order equations of (20)-(23) which in turn can be solved as (15), the linear dispersion relation for DASW's. This implies that linear dispersion relation for unmagnetized and magnetized situations are same. Magnetic field does not effect on the linear dispersion relation.

Equating the next higher order co-efficient of ε from above equations and by the use of parameters we can finally obtained ZK equation describing the nonlinear propagation of the DA SW's in the dusty plasma

$$\frac{\partial \phi^{(1)}}{\partial \tau} + PQ\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial Z} + \frac{1}{2}P \frac{\partial}{\partial Z} \left[\frac{\partial^2}{\partial Z^2} + R \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) \right] \phi^{(1)} = 0 \quad (34)$$

where,

$$P = \frac{(v_p^2 - \alpha \sigma_d)^2}{v_p} \quad (35)$$

$$Q = \frac{1}{2} \left[\mu_i - (\mu_e - \mu_p) \left(\frac{2-\gamma}{\beta^2} \right) \right] \quad (36)$$

$$R = 1 + \frac{v_p^4}{\omega_{cd}^2 (v_p^2 - \alpha \sigma_d)^2} \quad (37)$$

Equation (34) is a ZK equation for adiabatic DASW's with Maxwellian ions, degenerate electrons and positrons.

Figure 8: $P=0$ graph, showing the variation of β with μ_e and μ_p for the values of $\alpha=0.5$, $\mu_i=0.4$ and $\gamma=5/3$.

Figure 9: $P=0$ graph, showing the variation of β with μ_e and μ_p for the values of $\alpha=0.5$, $\mu_i=0.4$ and $\gamma=4/3$.

Figure 10: Variation of ϕ with ξ for the values of $\mu_p=1$, $\mu_i=0.4$, $\mu_e=1.2$, $\sigma_d=1$, $\omega_{cd}=0.3$, $\delta=20^\circ$, $\beta=0.7$, $u_0=0.1$, $\alpha=0.3$ (Red), $\alpha=0.4$ (Blue) and $\alpha=0.5$ (Black). Here dotted curve represented ultra-relativistic ($\gamma=4/3$) case and solid curve represented non-relativistic ($\gamma=5/3$) case.

Figure 11: Variation of ϕ with ξ for the values of $\alpha=0.5$, $\beta=0.7$, $u_0=0.1$, $\mu_p=1$, $\omega_{cd}=0.3$, $\delta=20^\circ$, $\mu_e=1.2$, $\sigma_d=1$, $\mu_i=0.4$ (Black), $\mu_i=0.5$ (Blue) and $\mu_i=0.6$ (Red). Here dotted curve represented ultra-relativistic ($\gamma=4/3$) case and solid curve represented non-relativistic ($\gamma=5/3$) case.

Solution of Zakharov-Kuznetsov Equation

The stationary solution (for simplicity, we have write $\phi^{(1)}$ as ϕ) of this ZK equation as,

$$\phi(\xi) = \phi_m \text{sech}^2(k\xi) \quad (38)$$

Where $\phi_m = [\frac{3u_0}{\delta_1}]$ is the amplitude and $k = \sqrt{\frac{u_0}{4\delta_2}}$ is the inverse of the width (Δ) of the SW's respectively; with $\delta_1 = PQ \cos \delta$ and $\delta_2 = \frac{1}{2} P(\cos^3 \delta + R \sin^2 \delta \cos \delta)$. δ being the propagation angle of SW's.

The $A=0$ ($P=0$) surface plots are shown for non-relativistic and ultra-relativistic case in absence (presence) of magnetic field are shown in figure 1 and 2 (8 and 9) respectively. From these figures we find that the SW's have positive potentials above the surface and negative potentials below the surface. Figure 3 shows the variation of ϕ with ξ for ultra-relativistic and non-relativistic cases. Figure 4, 5, 6 and 7 showing the variation of ϕ with ξ respectively for the different value of α , σ_d , β and μ_p . All of these figure shows the variation of amplitude and width for ultra-relativistic and non-relativistic cases in the absence of magnetic field. Figure 10, 11, 12, 13, 14 and 15 showing the variation of ϕ with ξ for the value of α , μ_i , δ , μ_e , σ_d and β respectively in the presence of magnetic field.

Figure 12: Variation of ϕ with ξ for the values of $\beta = 0.7$, $\alpha = 0.5$, $u_0 = 0.1$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1.2$, $\sigma_d = 1$, $\omega_{cd} = 0.3$, $\delta = 20^\circ$ (Red), $\delta = 30^\circ$ (Blue) and $\delta = 40^\circ$ (Black). Here dotted curve represented ultra-relativistic ($\gamma = 4/3$) case and solid curve represented non-relativistic ($\gamma = 5/3$) case.

Instability

We now study the instability of the opaquely propagating solitary waves, discussed in the previous section, by the method of small- k perturbation expansion (22-28). We first assume that separate ϕ as a function of ψ_0 and ψ as

$$\phi = \psi_0(Z) + \psi(\xi, \zeta, \eta, t) \quad (39)$$

Where ξ, ζ, η and t are components of new reference frame. For a long-wavelength plane wave perturbation in a direction with direction cosines (l_ξ, l_η, l_ζ) , ψ is given by

$$\psi = \phi(Z) e^{i[k(l_\xi \xi + l_\eta \eta + l_\zeta Z) - \omega t]} \quad (40)$$

in which $l_\xi^2 + l_\eta^2 + l_\zeta^2 = 1$, k is the wave constant, ω is the angular frequency of the waves and ϕ is the amplitude of the waves.

For small k , $\phi(Z)$ and ω can be expanded as

$$\phi(Z) = \phi_0(Z) + k\phi_1(Z) + k^2\phi_2(Z) + \dots \quad (41)$$

$$\omega = k\omega_1 + k^2\omega_2 + \dots \quad (42)$$

Doing some mathematical analysis we arrive at the following dispersion relation:

Figure 13: Variation of ϕ with ξ for the values of $\beta = 0.7$, $\sigma_d = 1$, $u_0 = 0.1$, $\omega_{cd} = 0.3$, $\delta = 20^\circ$, $\alpha = 0.5$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1$ (Red), $\mu_e = 1.1$ (Blue) and $\mu_e = 1.2$ (Black). Here dotted curve represented ultra-relativistic ($\gamma = 4/3$) case and solid curve represented non-relativistic ($\gamma = 5/3$) case.

Figure 14: Variation of ϕ with ξ for the values of $\beta = 0.7$, $\omega_{cd} = 0.3$, $\delta = 20^\circ$, $\alpha = 0.5$, $u_0 = 0.1$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1.2$, $\sigma_d = 0.6$ (Red), $\sigma_d = 0.8$ (Blue) and $\sigma_d = 1$ (Black). Here dotted curve represented ultra-relativistic ($\gamma = 4/3$) case and solid curve represented non-relativistic ($\gamma = 5/3$) case.

$$\omega_i = \Omega - l_\xi U_0 + (\Omega^2 - \Upsilon)^{1/2} \quad (43)$$

where

$$\Omega = \frac{2}{3}(\psi_m \mu_1 - 2\mu_2 \kappa^2)$$

$$\Upsilon = \frac{16}{45}(\psi_m^2 \mu_1^2 - 3\psi_m \mu_1 \mu_2 \kappa^2 - 3\mu_2^2 \kappa^4 + 12\delta_2 \mu_3 \kappa^4)$$

with,

$$\mu_1 = \delta_1 l_\xi + \delta_3 l_\zeta$$

$$\mu_2 = 3\delta_2 l_\xi + \delta_5 l_\zeta$$

$$\mu_3 = 3\delta_2 l_\xi^2 + 2\delta_5 l_\xi l_\zeta + \delta_6 l_\zeta^2 + \delta_7 l_\eta^2$$

It is clear from the dispersion relation (43) that there is always instability if $(\Upsilon - \Omega^2) > 0$. We can express the instability criterion as

$$S_i > 0 \quad (44)$$

with

$$S_i = l_\eta^2 [\omega_{cd}^2 + \sin^2 \delta] + l_\xi^2 \left[\omega_{cd}^2 - \frac{5}{3} (\omega_{cd}^2 + 1) \tan^2 \delta \right] \quad (45)$$

If this instability criterion $S_i > 0$ is satisfied, the growth rate $\Gamma = (\Upsilon - \Omega^2)^{1/2}$ of the unstable perturbation of these solitary waves is given by

Figure 15: Variation of ϕ with ξ for the values of $\alpha = 0.5$, $\omega_{cd} = 0.3$, $\delta = 20^\circ$, $u_0 = 0.1$, $\mu_p = 1$, $\mu_i = 0.4$, $\mu_e = 1.2$, $\sigma_d = 1$, $\beta = 0.6$ (Black), $\beta = 0.7$ (Blue) and $\beta = 0.8$ (Red). Here dotted curve represented ultra-relativistic ($\gamma = 4/3$) case and solid curve represented non-relativistic ($\gamma = 5/3$) case.

Figure 16: $S_i = 0$ surface plot showing the variation of ω_{cd} with l_η and l_ξ for the values of $\mu_e = 1.2$, $\mu_p = 1$, $\mu_i = 0.4$, $\beta = 1$, $\alpha = 0.5$, $\sigma = 1$ and $\delta = 20^\circ$.

$$\Gamma = \frac{2}{\sqrt{15}} \frac{U_0}{(\omega_{cd}^2 + \sin^2 \delta)} \sqrt{(\omega_{cd}^2 + 1) S_i} \quad (46)$$

Figure 16 represent $S_i = 0$ surface plot showing the variation of ω_{cd} with l_η and l_ξ . From this figure we found that ω_{cd} decreases with increasing the value of l_η very rapidly and increases slowly with increasing the value of l_ξ . Figure 17 showing the variation of growth rate Γ with σ_d and β . We found from this figure that Γ decreases with increasing the value of σ_d and β . Figure 18 shows the variation of Γ with μ_i and μ_e . We have found from this figure that Γ decreases with increasing the value of μ_e and μ_i . Figure 19 has shown the variation of growth rate Γ with α and β . We have seen from the figure that Γ increases with increasing the value of α .

Findings

DA SW's have been examined in a collisionless dusty plasma consisting of inertialess ultra-relativistic and non-relativistic electrons and positrons, inertial mobile positive dust particles, inertial Maxwellian ions by deriving K-dV equation using reductive perturbation technique. It has been found that the basic features of such DA solitary waves are significantly modified by the presence degeneracy of adiabaticness of components. The effects of the parametric regimes on solitary wave for both case of unmagnetized and magnetized fields are discussed below:

1. Depending on the value of μ_e , μ_p and β the solitary waves can be associated with positive (negative) potentials above (below) the $A = 0$ surface for unmagnetized and $P = 0$ surface for magnetized plasmas respectively.
2. The amplitude of the DASW's is higher for ultra-relativistic case than non-relativistic one.
3. Magnetic field enhance the amplitude of the SW's for both ultra-relativistic and non-relativistic case.
4. Both amplitude and width of the solitary waves increases with increasing the value of α , δ , σ_d and μ_e but decreases with increasing the value of μ_i , μ_p and β in the case of unmagnetized as well as magnetized dusty plasmas.

5. Cyclotron frequency ω_{cd} increases with increasing the value of l_η and decreasing the value of l_ξ .

6. The growth rate Γ decreases with increasing the value of σ , β , μ_e and but increases with increasing the value of α .

Figure 17: Variation of growth rate (Γ) with σ and β for the values of $\mu_e = 1.2$, $\mu_p = 1$, $\mu_i = 0.4$, $\delta = 2^\circ$, $\alpha = 0.2$, $l_\eta = 0.4$, $l_\xi = 0.5$, $u_0 = 0.1$ and $\omega_{cd} = 0.3$.

Figure 18: Variation of growth rate (Γ) with μ_i and μ_e for the values of $\alpha = 0.1$, $\beta = 0.7$, $\mu_p = 1$, $\omega_{cd} = 0.5$, $l_\xi = 0.5$, $l_\eta = 0.4$, $\delta = 15^\circ$, $\sigma = 1$ and $u_0 = 0.1$.

Figure 19: Variation of growth rate (Γ) with l_η and l_ξ for the values of $\beta = 0.7$, $\mu_p = 1$, $\omega_{cd} = 0.3$, $\mu_e = 1.2$, $\mu_i = 0.4$, $\delta = 2^\circ$, $\sigma = 1$, $l_\xi = 0.5$ and $u_0 = 0.1$.

Conclusion

In conclusion we can say that Maxwellian ions can modify the basic properties of the solitary waves significantly. On the other hand the presence of magnetic field effects the amplitude and width of the solitary wave and its instability. The properties of the solitary waves also moderated by the presence the adiabatic pressers in degenerate plasmas. We have shown the variation

of solitary wave phenomenon and its stabilities criterion as well as its growth rate due to the ultra-relativistic and non-relativistic degenerate pressure. These theoretical analysis may helpful astrophysicist to understand different critical situations exist in white dwarf and neutron stars; not only that, the other astrophysical objects where degenerate pressure plays a vital role to sustain these. Comparing the present analysis to others (non-degenerate objects) one give a prediction on the situation of degenerate astrophysical objects.

The plasma parameters used in the present investigations correspond to white dwarfs though it can also be applied for neutron stars. As the number density of plasma parameters, so as the value of β , is much higher for neutron stars than white dwarfs; it can be predict that the amplitude of the SW's would be higher. But numerical analysis is essential to view the true fact. We hope we will see the actual variation of wave properties between white dwarf and neutron stars very soon. We hope that our present investigation can helpful for understanding the white dwarfs and neutron stars and give a guideline for then who wants to work in the relevant field.

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