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Numerical Method for the Analysis of Thermal Radiation on Heat Transfer in Nanofluid

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Abstract

This present study investigates the effects of thermal radiation on heat transfer in nanofluid. The governing equations are formulated base on already existing model and are transformed to ordinary differential equations using stream function and similarity variables. The resulting dimension less equations are then solved numerically by Runge-Kuta Fehlberg method with shooting techniques using Maple software. With the help of graphs and tables, influences of the governing parameters are discussed. The result shows that the radiation has significance influences on heat transfer.

Keywords: Nanofluid; Runge Kuta-Fehlberg method; Shooting technique; Heat transfer; Radiation

Nomenclature

a = Constant; g = Acceleration due to gravity; k = Thermal Conductivity; P_r = Prandtl Number; T = Fluid Temperature; T_w = Surface Temperature; T_∞ = Free Stream Temperature; u,v = Velocity Components; x, y = Cartesian Coordinates; f(x) = Dimensionless Stream Function; G_r = Grashof Number; q_r = Heat Flux Radiation; B_o = Magnetic Field of Constant Strength; R = Radiation Parameter; K_s = Rosseland Mean Absorption Coefficient; K = Thermal Conductivity Coefficient

Greek Symbols

β = Thermal Expansion Coefficient; μ = Dynamic Coefficient of Viscosity; θ (η) = Dimensionless Temperature; η = Similarity Variable; ρ = Fluid Density; ψ = Stream Function; σ^* = Stefan-Bottzman Constant

Introduction

Nanofluid is formed when Nanoparticles such as Aluminium Oxide (Al_2O_3), Copper (Cu), Copper Oxide (CuO), Gold (Au), Silver (Ag), Silica particles, e.t.c are mixed with base fluids such as water, oil, acetone, ethylene, etc. The discovery of nanofluids in enhancing heat transfer in industrial processes has drawn the attention of both scholars and industrialists to make researches into this relatively new area.

Wang and Choi, [1] studied the thermal condutivity of nanoparticle fluid mixture containing Al_2O_3 and CuO nanoparticles and showed that the thermal conductivity of nanofluids increased with increasing volume fraction of the nanoparticles. Steve (2006) found out that a small size radiator filled with well prepared nanofluid containing monosized nanoparticles with 2nm nominal diameter will be okay for heavy vehicle instead of a very big radiator and this will save cost and space. And he also discovered that fuel efficiency will be reduced by 5% in these Heavy Trucks when nanofluids are introduced into their cooling system. John and Baldev, [2] carried out experiment and they concluded that by controlling the linear aggregation length from nano- to macron scales, the thermal conductivity of the nanofluid can be tuned from a low to very high value and that under repeated magnetic cycling, the thermal conductivity is reversible.

Khan and Ipop, [3] studied the boundary-layer flow of a nanofluid past a stretching sheet and their result shows that the heat transfer at fixed values of the Lewis number (*Le*), the Brownian motion parameter (*Nb*) and the thermophoresis parameter (*Nt*) increases with the Prandtl number (*Pr*). This is because a higher *Pr* fluid has relatively lower thermal conductivity which reduces conduction and thereby increases the heat transfer rate at the surface of the sheet.

Godson, et al. [4] in separate papers agreed that Nanofluids are the most option to enhance heat transfer and showed that a very small amount of nanoparticles suspended stably in base fluids can provide impressive improvement in the thermal property of such fluids.

Zeinali and Salehi, [5] studied experimentally, the effect of magnetic field of various strengths on the thermal performance of Silver/water nanofluid and their results showed that the thermal efficiency in the presence of magnetic field increased.

Yohannes and Shankar, [6] examine the boundary layer flow of heat and mass transfer in MHD flow of nanofluids through porous media with viscous dissipation and chemical reaction. The governing boundary layer equations were formulated and

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reduced to a set of ordinary differential equations using similarity transformations and then solved numerically by an explicit finite differential scheme known as the Keller box method. Wan-Mohd, [7] applied Runge-kutta Fehlberg method and shooting technique for solving classical Blasius equation [7]. The well-known Blasius equation is governed by the third order nonlinear ordinary differential equation and then solved numerically using runge-kutta Fehlberg method with shooting techniques. Hamad, [8] found analytical solution to the differential equations that model heat transfer in nanofliud. We extended the work of Hamad, [8] by incorporating radiation term to have differential equations that model the effect of thermal radiation on heat transfer of a nanofliud.

Mathematical formulation

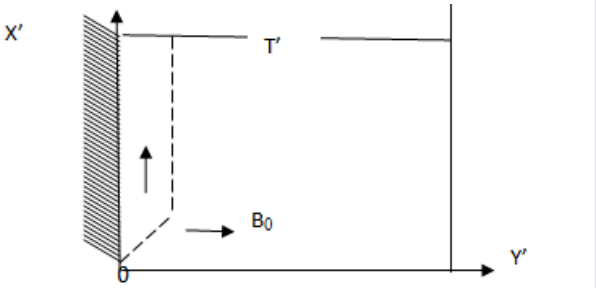

Figure 1

Consider the steady laminar two-dimensional flow of an incompressible viscous nanofluid past a linearly semi-infinite stretching sheet under the influence of a constant magnetic field of strength B_0 which is applied normally to the sheet. $x'$and y' are the coordinates along and perpendicular to the sheet while u' and v' are the velocity components in the directions respectively as shown in the figure below:

A water based nanofluid containing different types of nanoparticles: Al_2O_3, Cu, TiO_2 and Ag is used with the assumption that both the fluid and the nanoparticles are in thermal equilibrium. Based on the already existing model of Hamad, [8], the governing boundary layer equations of continuity, momentum and heat equations formulated are:

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{1}$$

$$\rho_{nf}\left[u'\frac{\partial u'}{\partial x'}+v'\frac{\partial v'}{\partial y'}\right]=\mu_{nf}\frac{\partial^2 u'}{\partial y'^2}-\sigma B_0 u'+g\beta_t\left(T'-T'_\infty\right) \tag{2}$$

$$(\rho c_p)_{nf}[u'?\partial T'/\partial x'+v'?T'/(?y')]=K_{nf}(\partial^2 T')/\partial y'^2-(\partial q_r)/\partial y' \tag{3}$$

The boundary conditions of the equations are

$$u'=u'_w(x')=ax', v'=0, T'=T'_w T'_w aty'=0$$

$$u'\to 0, T'\to T'_\infty, y'\to\infty \tag{4}$$

Where q_r is the radiative heat flux, T' is the temperature of the fluid, $x'$and y' are the co-ordinates along and perpendicular to the sheet while u' and v' are the velocity components in the x' and y' directions respectively and a is a constant.

The effective density (ρ_{nf}), effective dynamic viscosity (μ_{nf}), heat capacitance $(\rho C_p)_{nf}$ and the effective thermal conductivity (k_{nf}) of the nanofluid, in that order, are given as

$$\begin{aligned}\rho_{fi} &= (1-A)\rho_f + A\rho_s\\ \mu_{fi} &= \frac{\mu_f}{(1-A)^{2.5}}\\ (\rho C_p)_{fi} &= (1-A)(\rho C_p)_f + A(\rho C_p)_s\\ k_{fi} &= k_f\left\{\frac{k_s+2k_f-2A(k_f-k_s)}{k_s+2k_f+2A(k_f-k_s)}\right\}\end{aligned} \tag{5}$$

Where A is the solid volume fraction (A ≠ 1), μ_f is the dynamic viscosity of the base fluid, while ρ_f and ρ_s are the densities of the pure fluid and the nanoparticle respectively. The constants k_f and k_s are the thermal conductivities of the base fluid and the nanoparticle respectively.

Following Rosseland approximation Brewstar [9] the radiative heat flux q_r is modeled as

$$q_r=\frac{4\sigma'}{3k'}\frac{\partial T'^4}{\partial y'} \tag{6}$$

Where σ' is the Stefan-Boltzman constant and k' is the mean absorption coefficient. Assuming that the difference in temperature within the flow is such that T'^4 can be expressed as a linear combination of the temperature, we expand T'^4 in Taylor's series about T_∞ as follows:

$$T'4=T_\infty'^4+4T_\infty'^3(T'-T'_\infty)+6T_\infty'^2(T'-T\infty')^2+\cdots \tag{7}$$

and neglecting higher order terms beyond the first degree in $(T'-T'_\infty)$, we have

$$T'^4\approx -3T_\infty^{;4}+4T_\infty'^3T' \tag{8}$$

Differentiating equation (6) with respect to y and using equation (8) to obtain

$$\partial q_r)/\partial y')=-16T_\infty^3\sigma')/3k'-\partial^2T'/\partial y'^2 \tag{9}$$

Then equation (3) becomes

$$\left(\rho c_p\right)_{nf}\left[u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right]=K_{nf}\frac{\partial^2 T'}{\partial y'^2}+\frac{16T_\infty^3\sigma'}{3k'}\frac{\partial^2 T'}{\partial y'^2} \tag{10}$$

By the introduction of the following variables

$$u=u'/\sqrt{av_f}\,v=v'/\sqrt{av_f}\,\theta=(T'-T'\infty/$$
$$(T'w-T'\infty), x=x'/\sqrt{(v_f/a)}.y=y'/\sqrt{v_f/a} \tag{11}$$

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Equation (11) transform equation (1), (2) and (10) to the followings

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{12}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{(1-A)\rho_f+\rho_s}\left(\frac{1}{(1-A)^{2.5}}\frac{\partial^2 u}{\partial y^2}-M+G_r\theta\right) \tag{13}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{p_r}\frac{1}{(1-A)(\rho C_p)_f+A(\rho C_p)_s}\left(\frac{k_{fi}}{k_f}+R_d\right)\frac{\partial^2\theta}{\partial y^2} \tag{14}$$

Where $M=\frac{\sigma\beta_0}{a}$ is the magnetic field parameter, $p_r=\frac{v_f}{K_f}$ is the Prandtl number, $R_d=\frac{16\sigma' T_\infty'^3}{3K^* K_f}$ is the radiation parameter,

$G_r=\frac{g\beta_t\left(T_w'-T_\infty'\right)x}{a}$ Is the Grashof number

And the boundary conditions becomes

$$\left.\begin{aligned} u=x, v=0, \theta=1 \text{ at } y=0 \\ u\to 0, \theta\to 0 \text{ as } y\to\infty \end{aligned}\right\} \tag{15}$$

By introducing the stream function ψ, which can be defined as

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x}.$$

and using the similarity transformations

$$\eta=y, \psi=f(\eta), \theta=\theta(\eta) \tag{16}$$

We have

$$f'''+(1-A)^{2.5}\left\{ff''-(f')^2\right\}\left[(1-A)\rho_f+\rho_s\right)-(Mf'+G_r\theta)\right]=0 \tag{17}$$

$$\frac{1}{P_r}\frac{1}{(1-A)(\rho C_p)_f+A(\rho C_p)_s}\left[\frac{k_{fi}}{k_f}+R_d\right]\theta''(\eta)+f(\eta)\theta'(\eta)=0 \tag{18}$$

And the boundary conditions now becomes

$$\left.\begin{aligned} f(0)=0,\ f'(0)=1 \text{ at } \eta=0 \\ f\to 0 \text{ at } \eta\to\infty. \\ \theta(0)=1 \text{ and } \theta(\infty)=0, \end{aligned}\right\}$$

Table 1: Thermo physical properties of water and nanoparticles Hamad [8].

Compound	ρ (kg/m³)	C_p (J/kgK)	k (W/mK
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Alumina (Al_2O_3)	3970	765	40
Silver (Ag)	10500	235	429
Titanium Oxide (TiO_2)	4250	686.2	8.9538

An algorithm was written and run with Maple codes to plot graphs and generate tables for analysis.

Numerical Solution

The thermo physical properties of pure water and those of the nanoparticles as given in table 1 by Hamad [8] will now be substituted in the transformed coupled ordinary differential equations. These equations will now be solved using Runge-Kutta-Fehlberg (RKF) method with shooting technique. The algorithm of Runge-Kutta-Fehlberg (RKF) method is order 5 and it gives a better approximation of the solution than the general Runge – Kutta method of order 4.

An algorithm was written and run with Maple codes to plot graphs and generate tables for analysis.

Results

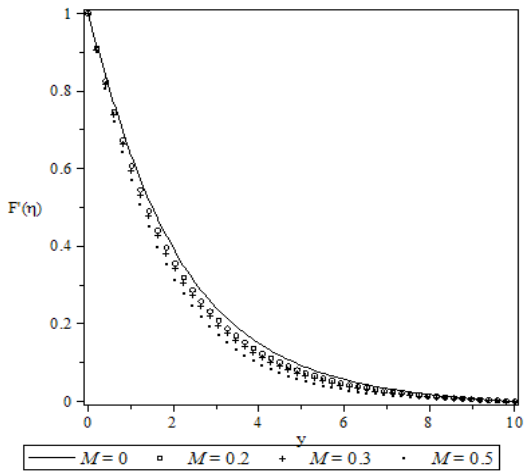

Figure 2: Effect of M on velocity distribution profiles.

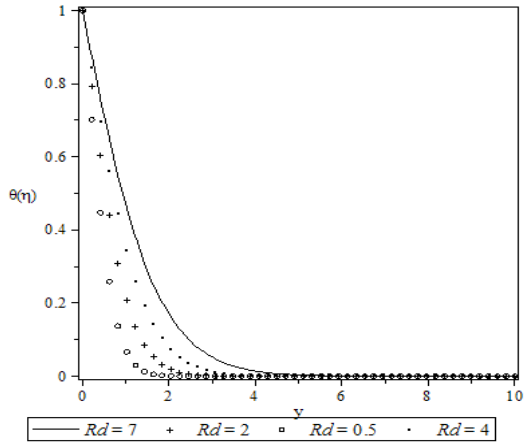

Figure 3: Effect of Rd on Temperature distribution profiles.

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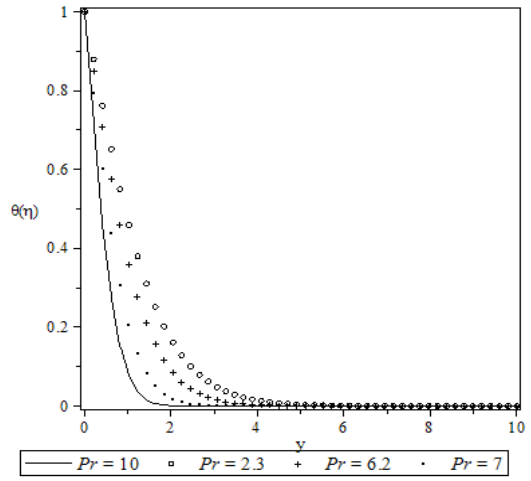

Figure 4:

Table 2: Values of $-\theta'(0)$ for various Rd, A When $Gr = 0.1$, $M = 0.5$ and $\Pr = 6.2$.

Rd	A	$-\theta'(0)$			
		Cu	*Ag*	Al_2O_3	TiO_2
0.5	0.30	1.31430	1.23109	1.28489	1.28961
	0.35	1.33022	1.23834	1.29732	1.30086
	0.40	1.34938	1.24650	1.31206	1.31409
	0.45	1.37892	1.25349	1.33263	1.33158
2	0.30	0.85353	0.79483	0.83119	0.82915
	0.35	0.86695	0.80360	0.84275	0.84011
	0.40	0.87137	0.80646	0.84654	0.84371
	0.45	0.87576	0.80929	0.85030	0.84727
4	0.30	0.60425	0.55941	0.58668	0.58351

Results are shown in figures 2- 3 and table 2

From derivation, the the Nuselt number can be written as follows;

$$Nu = -\theta'(0)$$

Discussion of Results

We have formulated the differential equations modeling the effect of radiation on heat transfer in nanofluid based on the already existing model. Numerical evaluation of the numerical solutions reported in the previous sections was performed and the results are presented in graphical and tabular forms. This was done to illustrate the influence of some parameter involved.

The effect of magnetic field parameter on velocity distribution profiles across the boundary layer is presented in Figure 1. It is obvious that the effect of increasing values of the magnetic field parameter M results in a decreasing velocity distribution. This is due to the fact that the introduction of transverse magnetic field normal to the flow direction has a tendency to create a drag force due to Lorentz force and hence results in retarding the velocity profiles Figure 2 shows the influence of the radiation parameter on the temperature distribution profiles. The temperature increases as the radiation increases. Figure 3 presents the temperature distribution profiles for different values of the Prandtl number (Pr). The results show that the effect of increasing values of the Prandtl number results in a decrease in the temperature.

In table 2, as the radiation parameter increases, the nuselt number of the nanoparticles decreases, that is to say the effect of increasing the radiation parameters is to decrease the rate of heat transfer of the nanoparticles.Also, as the fraction volume parameter (A) increases, nuselt number increases.

Conclusion

Numerical solution was applied to the differential equation that model the effect of thermal radiation on heat transfer on nanofluid using runge-kutta Fehlberg method. It was found that the effect of high radiation is to decrease the rate of heat transfer.

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