Modeling and Simulation of a New Integrated Electrohydraulic Actuator for Humanoid Robots

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Abstract

The work presented in this paper is an important step toward a better understanding of a compact hydraulic robotic actuator, based on the Integrated Electro-Hydraulic Actuator (IEHA) developed by Alfayad and Ouezdou [1]. The novel advantage of this actuator is being highly compact and autonomous (no need for central hydraulic source), while keeping a good power to weight ratio. In order to present and develop the working dynamics of this actuator, a highly detailed mathematical model for the system is presented. The proposed model is simulated using MATLAB-Simulink software to identify the effect of the internal system parameters on system dynamics and prepare an input-output test-bed model. Such test-bed model is used to obtain the transfer function of the system and its order. Analysis of the effects of the main parameters was carried out and a lower order of the system was identified. A linear model of the system is derived and validated using system identification technique. Finally, a robust motion controller is applied on the proposed linear model and the simulation results are presented.

Keywords: Hydraulic Actuation; Mathematical Modeling; System Identification Technique; Virtual Modeling;

Nomenclature

Cd: The venta contracta coefficient [-]CHeA & CHeB; The two sides of the carriage moving w.r.t the fixed frame [-]; d: Distance between the bottom of chamber and The center of the shaft [mm]; E: Eccentricity of the integrated actuator [mm]; Fp: The force exerted by the oil pressure in the Radial pump on the carriage [N]; Fe: The hydraulic force in the carriage chambers [N]; Fext: The external force exerted by the cylinder [N]; Fc: The hydraulic force exerted on the cylinder [N]; Hp1: The distance between the pistons of length l_p at the dead bottom position and the bottom of its chamber [mm]; Hp2: The distance between the pistons at the high Dead point and the bottom of its chamber [mm]; Hpi: The height of oil in piston chamber (i) at each Instant [mm]; Kin: Internal leakage coefficient [-]; Kout: Internal leakage coefficient [-]; Lp: The stroke length of the piston [mm]; Lpi: The distance between the shaft center + Piston contact point on the surface of the Housing [mm]; m: The end effectors mass [kg]; me: Mass of the carriage of the IEHA [kg]; N: Number of micropistons [-]; Prp: The pressure of the oil at the intake channel of the radial pump [bar]; Pc: The pressure difference between the two Cylinder chambers A and B [bar]; Ps: High pressure line [bar]; Pa: Pressure in chamber A of the micro-pistons [bar]; Pb: Pressure in chamber B of the micro-pistons [bar]; Pi: The micro-pistons of the IEHA [-]; Qmac: The average macroscopic flow of the N Micro-pistons [m3/s]; Qmic: The average microscopic flow of the N Micro-pistons [m3/s]; Qe: The flow from micro-valve into the carriage Chambers [m3/s]; Qeleak: The leakage between the carriage and output Cylinder + the one between the hydraulic Chamber to the micro pumps body [m3/s]; Qpleaf: The leakage flow of the micro-pistons in the Radial pump [m3/s]; Qleaf: The internal leakage between the two Chambers of the cylinder [m3/s]; Rb: The radius of the carriage [mm]; rtig: The micro-valve radius [mm]; rpp: The radius of the in-out opening section of the micro-pump [mm]; Sc: The surface area of the linear hydraulic Cylinder’s piston [cm2]; Se: The surface area of the carriage chambers [cm2]; Spi: The active area of a single piston [mm2]; V: Fluid volume [m3]; ve: The volume of the chamber of the carriage [m3]; vpi: The volume of the IEHA micro-pistons [m3]; X: Micro-valve input displacement [mm]; Y: The end effector (output load cylinder) Position [mm]; β: Bulk modulus of elasticity [MPa]; θ: The angle between the piston and the Reference axe [degrees]; ω: Rotational speed of the shaft [rad/sec]; ρ: The density of the oil [kg/m3]; ζ: The actual pressure in the piston chamber [bar]; ϕi: Phase angle of the piston pi [deg]

Introduction

The development of bio-inspired robots; an important and
active area of research; has been going on for the past few years. These robots outperform the mobile robots in terms of mobility and versatility [2]. Moreover, they combine many desirable features such as human-like locomotion capabilities and human-friendly design and behavior. However, the performances exhibited by bio-inspired robots are more or less limited. This is mainly due to their actuation. Indeed, in our opinion, the first still an open question that should be answered while designing a bio-inspired robot concerns its actuation.

Research in the actuation of bio-inspired robots that aims to mimic the performances of biological systems, has been investigated by several research teams. Nevertheless, as far as we know, no actuator able to reproduce the biological muscle capabilities in term of producing force and speed already exists. Focusing on humanoid robots, a continuous need for enhancing their performances leads to identifying the desired actuator properties. These properties are: i) high power to mass ratio; ii) high integration within the robot body; iii) safe-interaction of the humanoid with the surrounding environment while performing human-like behavior.

Basically, the actuation for humanoid robots can be either electrically or hydraulically. Most robotic applications are electrically driven. Generally, electric motors with high gear ratio drives are popular because of their small size and cheap price. In addition, electric motors are proven to be easy to use and control. Significant examples of electrically actuated robots are: ASIMO [3], ROBIAN [4], HRP biped series [5], Johnnie and LOLA [6], REEM [7].

However, electric actuation has several drawbacks. Indeed, electric motors normally produce small torques relative to their size and weight, thereby making reduction sub-systems with high ratios essential to convert velocity into torque. These reduction components are limited and cannot increase indefinitely, which resulted in having reduced dynamic capabilities systems. Moreover, the presence of high reduction ratios causes limited passive back-drivability, which may lead to unsafe interaction with humans as well as troubles for walking in unforeseen terrains [8].

Nevertheless, several research works to enhance the performance of electric actuators for humanoid robots were proposed. For instance, flexible elements were added between the motor and the load, as can be seen in ECD leg [9]. Tekken [10] in order to have locomotion activities over unforeseen terrain. Other robots, like OGG [11] and DOMO use series elastic actuators developed by Robinson et al [12]. In this kind of robots, robustness of the system is an issue because of complex electric connection [13]. On the other hand, harmonic drives were also utilized for example in ARMAR III [14]. In this robot, motors and the harmonic drives are located in the thorax of the robot. This design intended to decrease the weight of the arm. However, this inherently leads to complex transmission system through wires, which also reduces the robustness.

In general, electric actuators coupled with gearbox reduction systems currently didn’t fulfill all the needs of humanoid robots. Indeed, this actuation solution neither has a high power-to-weight ratio nor is able to simultaneously provide the speed and forces required for highly dynamic robots. Therefore, other methods of actuation are commonly sought after.

The other concurrent actuation solution is hydraulic technology based. It has several advantages, which mainly include: 1) high power to mass ratio; 2) ability to produce high torque at low speed; 3) the high stiffness compared to electric ones; 4) the ability to perform continuous, intermittent, reversing and stalled motions without damage; 5) the ability to emulate human musculo-skeletal systems by means of high-bandwidth force control. Examples of robots using hydraulic actuation include: Bigdog [15], Sarcos [16], HyQ [17], Tae-Mu [18] and the underdevelopment humanoid robot HYDROID [19].

However, this type of actuators has its main drawbacks. The major one is due to the necessity of a central Hydraulic Power Unit (HPU) to supply high-pressure fluid to all the robot joints. This HPU is always bulky and the leakage from the hydraulic tube connections can cause safety issues, especially in human-robot interaction. Hence, it was necessary to have integrated hydraulic actuators inside the robot and near the joints.

Research for hydraulic actuator for robotics has been started with Bobrow et al [20], in which a closed loop hydrostatic actuator was introduced. This actuator was driven directly from an electric motor without a gear train enabling large speed reductions and corresponding torque amplification. However, achieving high torques is not possible without the use of large electric motors and power amplifiers, which leads to an increased overall dimensions and large mass for the system. Moreover, this actuator suffered from dead band caused by the inversion of electric motor rotation. S. Habibi et al. followed [21], with an improvised electrohydraulic actuator that tackled the effect of dead band by using a high gain cascaded control strategy with motor speed feedback. Nevertheless, this actuator had several design constraints to achieve high performance. These constraints include the usage of a symmetrical actuator in addition to appropriate sizing of hydraulic components to minimize the pressure drop. These constraints questioned the supposed compactness of an electrohydraulic actuator dedicated for robotic applications.

Recent research was done on the EHA compactness by Gnesi et al [22] and Takahashi et al [23], in which both presented two EHAs made for aircraft applications. The first presented an EHA with a vane pump and double acting cylinder, while the latter used a similar design but with a piston pump. The most recent contribution was attributed to Altare et al [24] and [25], in which a miniature gear pump is presented along with its actuator. The volumetric displacement of this EHA was found to be 0.13 cc/rev and it was also, made solely for aerospace applications.

In 2011, an IEHA (Integrated Electro-Hydraulic Actuator) was developed by Alfayad et al. [26] and [31]. Its main objective was eliminating the need for a central pressure source and to be implemented for each joint of the humanoid hydraulic robot HYDROID. Due to the different pressures needed by the robot
The main objective for the IEHA is to be implemented on the humanoid robot HYDROID. Hence, the human robot interaction and compliance are of high priority. There are different ways to ensure that the robot does not risk injuring the user. One approach to soft human robot contact is back-drivability. This ability enables the mechanical system to move the input axis from the output axis. In other words, the force applied from output axis of the actuator must be greater than force lost in the actuator due to static friction [27, 28]. A more advanced approach is to apply an active compliance, which can be reached through accurate force/position control [29]. Both of these approaches need a complete dynamic model of the actuation system, including the IEHA actuators and the several transmission mechanisms.

Consequently, the goal of the work presented in this paper concerns the study the internal parameters of the IEHA which highly complex and therefore identify their influence on the behavior of the whole system. The ultimate goal is deducing a practical dynamic model that approximates the IEHA behavior for active compliance control targeting. Indeed, the dynamic model will be used to choose the best control strategy to ensure the back-drivability. Moreover, the identification of the internal parameters in the literature was done in most cases through empirical assumptions and experimental results. Due to the high compactness of this kind of actuators, there are different ways to ensure that the robot does not risk injuring the user. One approach to soft human robot contact is back-drivability. This ability enables the mechanical system to move the input axis from the output axis. In other words, the force applied from output axis of the actuator must be greater than force lost in the actuator due to static friction [27, 28]. A more advanced approach is to apply an active compliance, which can be reached through accurate force/position control [29]. Both of these approaches need a complete dynamic model of the actuation system, including the IEHA actuators and the several transmission mechanisms.

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Meanwhile, an analytical study of the equations is carried out to define an enhanced input-output relation of the system. Finally, a system identification technique is used to validate the linearized model.

This paper is organized as follows; section 4 briefly describes the architecture of the IEHA electro-hydraulic actuator, IEHA. Section 5 introduces the fundamental hydraulic and mechanical equations used in the development of the virtual model of the actuator. In Section 6, the virtual model of the system under MATLAB-Simulink is developed and the influence of the internal parameters is studied. A mathematical analysis of the input-output relation of the system is presented in Section 7. In Section 8, a linearized model of the system is achieved through black-box system identification. This linear model is used in Section 9 to implement a predictive position controller for the actuator. The conclusion and the future work are shown in Section 10.

**The Architecture of the IEHA**

The integrated hydraulic actuator (IEHA) is based on the power transmission from an electric motor to a hydraulic actuator. The basic idea consists of converting the electric power to mechanical one by using a highly integrated micro-pump producing pressure and flow (shown in Figure 2). This energy converter contains an in-built micro-valve that controls the eccentricity. The radial pump is connected to the output piston (linear or rotary) through a passive distributor. In order to simplify the figure, only two of the pump’s micro-pistons are shown. Since the passive distributor response is very fast compared to the rest of the components, its dynamics can be neglected in the present study. In this section, the functioning principle of the micro-pump and the micro valve are detailed.

**The Radial Micro-Pump**

To produce hydraulic energy in the authors proposed solution [1], a micro radial pump is used to deliver the hydraulic power required. The flow discharge of such pump can be controlled by modifying the eccentricity of the main shaft driven by the electric motor with respect to the housing. For a given direction of rotation of shaft, fluid enters the pump, and the centrifugal forces and hydraulic pressure push the micro-pistons to the walls of the housing during half of the rotation cycle, [0, π], where the corresponding micro-pistons are connected to the intake port. As the rotor continues around, the vanes sweep the fluid to the opposite side. For the other half of the cycle [π, 2π], the micro-piston volume decreases and fluid exits the discharge port. Figure 3 presents a simplified IEHA model with two micro-pistons. The distance between the bottom of chamber and the center of the shaft is denoted d, while Rb is the radius of the carriage. Hp1 is the distance between the micro-piston of length lp at the dead bottom position and the bottom of its chamber. In the same way, Hp2 is the distance between the micro-piston at the high dead point and the bottom of its chamber as given in Equations 1 and 2. The piston stroke is defined by the volume of fluid produced during a rotation for a given eccentricity (E). In order to calculate
the variation of the micro-pump stroke, the distance that the piston travels during half-rotation corresponding to either the intake or the discharge is determined.

\[ H_{p1} = R_b - E - l_p - d \]  
\[ H_{p2} = R_b + E - l_p - d \]  

Micro-valve

Moving the carriage from one position to another between the two extreme values of the eccentricity, \( E_{\text{max}} \), changes the micro-piston stroke and the flow produced by the micro-pump. The micro-valve has the role of adjusting the value of the eccentricity. For the latter, as shown in Figure 2, two simple effect jacks CHeA and CHeB are integrated on the sides of the carriage to move it with respect to the fixed frame.

By activating the voice-coil of the micro-valve in positive or negative direction, CHeB is connected to high pressure fluid line Ps, while CHeA is connected to the return line, and vice versa. In order for the eccentricity to follow the value of input displacement X, a closed loop system is needed. This closed loop circuit has been carried out mechanically, by connecting the micro-valve external fixed part to the pump housing. Therefore, when X changes, the micro-valve is opened and E changes until it reaches the value of X, where the micro-valve closes and eccentricity remains constant at this value. An external force applied to the actuator increases the pressure in the output piston, and consequently the micro-pump pistons. This increase in pressure, increases the force applied by the micro-pistons on the housing, and can change the eccentricity. In this case the micro valve opens which will correct the eccentricity and bring it back to the same value as X.

The Dynamic Equations of the IEHA

In this section, the necessary hydraulic and mechanical equations of the dynamic model are presented. In addition to the leakage, the compressibility parameters and the microscopic flow equations of the micro radial pump are taken into account. These equations are based on the prototype of IEHA where the micro radial pump delivers flow from a high-pressure supply to one side of the cylinder at each instant. The other side of the cylinder is connected to an atmospheric reservoir. The way the micro radial pump is connected to the output cylinder (chamber A or B) depends on the sign of the eccentricity and is done by the passive distributor. The dynamics of the passive distributor are not taken into account, because it has a very fast response time compared to the rest of the system.

Fundamental Hydraulic Equations

(a) The radial micro-pump

Each micro-piston in the radial pump is connected to the intake line during half of each rotation, where it sucks oil. During the second half, it pushes oil to one of the two cylinder chambers. The micro-pistons are pushed to the carriage wall by centrifugal forces. The microscopic model of flow is:

\[ Q_{pi} = \begin{cases} S_{pi} H_{pi} & \theta \in [0, \pi] \\ -S_{pi} H_{pi} & \theta \in [\pi, 2\pi] \end{cases} \]  

\[ Q_{\text{mic}} = \sum Q_{pi} \]  

Where \( S_{pi} \) is the active area of a single micro-piston, and \( H_{pi} \) is the time derivative of \( H_{pi} \), the height of oil in micro-piston chamber i at each instant. \( H_{pi} \) is calculated through a geometrical study of the movement of the micro-piston inside the housing. To make the calculation simpler, we define \( L_{pi} \), as the distance between the shaft center and the micro-piston contact point on the surface of the housing, shown in Figure 4.

\[ L_{pi} = d + H_{pi} + l_p \]

As shown in Figure 4, \( \theta \), the angle between the micro-piston and the reference axe, is a function of time and the rotational speed of the shaft \( \omega \) and equals \( \omega t + \phi_i \). Using the Pythagorean Theorem, \( L_{pi} \) is calculated as a function of eccentricity:
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\[ p = -\beta \frac{\dot{V}}{V} \]  

(10)

Where \( P \) and \( V \) represent fluid pressure and volume, respectively. Higher values of \( \beta \) make the hydraulic system stiffer, and easier to control. However, in robotic applications, especially in human-robot interaction, it is not desirable to have very stiff actuation, due to safety issues. This leads to increase in response time, and may make the system unstable. These undesirable phenomena can be compromised with a good choice of the other system variables (such as the number of micro-pistons, or maximum value of eccentricity), combined with an efficient control strategy. Compressibility has a spring-like effect on the movement of the micro-piston, and is taken into account at all levels of the IEHA design.

(b) Hydraulic Oil Leakage

In contrarily to compressibility, leakage has a damping effect. In carriage and output cylinder, this leakage is from the high-pressure chamber to the low pressure one. In the micro-pistons, the leakage takes place from the hydraulic chamber to the micro pump body. In the latter, this leakage inhibits a large increase in fluid pressure, when connected to the intake channel. The leakage flow \( Q_{\text{leak}} \) is given by the following relation:

\[ Q_{\text{leak}} = K_{\text{in}}(P_a - P_h) + K_{\text{ex}}(P_i) \]  

(11)

Where \( K_{\text{in}} \) and \( K_{\text{ex}} \) are the internal and external leakage coefficients respectively. These two values are a function of the surface of the leakage section.

Motion Dynamics

(a) Carriage Eccentricity

The dynamic equation of the carriage, using eccentricity \( E \) can be written as:

\[ F_e + F_p = m_e \ddot{E} \]  

(12)

Where \( F_p \) is the force exerted by the oil pressure in the radial pump on the carriage, while \( F_e = 2 Se \xi \) is the hydraulic force in the carriage chambers. \( 2\xi \) is the pressure difference between its two sides, which changes as a function of the compressibility and the real flow \( Q_e \) in the carriage which is given by:

\[ Q_e = S_e \dot{E} + \frac{V}{\beta} \xi - Q_{\text{leak}} \]  

(13)

Where \( Q_{\text{leak}} \) is the leakage from the carriage micro-pistons to the body of the actuator. \( v \) and \( \beta \) are the chamber volume and the bulk modulus of the oil respectively.

(b) Fluid pressure in the radial micro-pump

For a given eccentricity, one side of the pump is directly connected to the supply terminal from the micro-valve, and the

\[ L_{pi} = \sqrt{R_i^2 - E^2 \sin^2 (\omega t + \varphi_i) - E \cos (\omega t + \varphi_i)} \]  

(5)

By taking the time derivative, the speed of micro-piston \( i \) is obtained as:

\[ H_p = L_p = -\dot{E} \cos (\omega t + \varphi_i) + E \omega \sin (\omega t + \varphi_i) - \frac{E E \sin^2 (\omega t + \varphi_i) + E \omega \sin (\omega t + \varphi_i) \cos (\omega t + \varphi_i)}{\sqrt{R_i^2 - E^2 \sin^2 (\omega t + \varphi_i)}} \]  

(6)

Using the data for the minimum and maximum chamber volume, according to Equations 1 and 2, it can be seen that in each rotation, one single micro-piston delivers an amount of oil equal to:

\[ (H_{p2} - H_{p1})S_{pi} = 2ES_{pi} \]  

(7)

Hence, the average macroscopic flow of \( N \) micro-pistons can be expressed as:

\[ Q_{\text{mac}} = 2NS_E \omega \]  

(8)

(b) Micro-valve

The flow from the micro-valve into the carriage chambers follows the Bernoulli's equation and is a function of the instant opening displacement of the valve \( (X(t) - E(t)) \), and the actual pressure \( \zeta \) in the chamber:

\[ Q_e = C_d 2\pi r_{tg} (X - E) \sqrt{P \sqrt{\frac{2}{\beta}} - \zeta} \]  

(9)

Where \( C_d \) is the vena contracta coefficient, \( r_{tg} \) is the micro-valve radius, while \( P \) is the oil density, and \( Ps \) is the high-pressure supply line.

Additional hydraulic parameters

(a) Compressibility of Hydraulic Oil

The oil compressibility is defined as the relative change in oil volume per unit change in pressure. Oil compressibility should be taken into account when response time and high-precision control of hydraulic actuators are important. The resistance of a fluid to being compressed is defined by the variable, Bulk Modulus \( (\beta) \), which is the inverse of compressibility as shown in Equation 10.
other side of the micro-pump is connected to one chamber of the output actuator. Each micro-piston is connected to the supply line in the first half of its rotation, where it aspirates fluid. In the second half, it ejects the fluid to the cylinder chamber. However, in the presence of piston leakages, Qpileak, the pressure increase in the micro-pistons is not very large. For each micro-piston Pi we can consider:

\[ C_i \Pi \left( \frac{\partial P}{\partial t} \right) = Q_i + \frac{\partial v_i}{\partial \theta} - Q_{lieak} \]  

(14)

With,

\[ \Delta P_i = \begin{cases} \frac{P_i - P_{\beta}}{\theta \in [0, \pi]} & \text{if } \theta \in [0, \pi] \\ \frac{P_i - P_{\beta}}{\theta \in [\pi, 2\pi]} & \text{if } \theta \in [\pi, 2\pi] \end{cases} \]  

(15)

where \( r_{rp} \) is the radius of the in-out opening section of the micro pump intake tube, \( v_{pi} \) is the volume of Pi, \( P_{rp} \) is the pressure of the intake channel of the pump, and \( \lambda \in \{ A, B \} \) represents chamber A or B of the output cylinder, and depends on the sign of the eccentricity. In addition to \( \beta \), \( Q_p \), \( C_d \), \( P \) and \( Q_{pileak} \) which are bulk modulus of elasticity, flow inside micro-piston, venta contracta constant, density of the oil and internal leakage of the micro-piston respectively. The fluid pressure in micro-pistons exerts a force on the carriage that can be expressed as:

\[ F_p = (P_{rp} - P_{\beta}) S_p \cos(\omega t) \]  

(c) Position of the linear actuator

The dynamic equation of the hydraulic linear actuator is given by Equation 17, where \( F_{c, ext} = \) hydraulic force and the external force exerted on the cylinder respectively. \( M \) and \( Y \) are the end-effector mass and position respectively. \( 2P_{c} \) is the pressure difference between the two cylinder chambers A and B and Sc is the surface area of the linear hydraulic piston cylinders.

\[ F_{c} - F_{ext} = m \ddot{Y} \]  

(17)

Pressure change due to compressibility and the input flow is defined as:

\[ Q = S \dot{Y} + \frac{\dot{V}}{\beta} P_{c} - Q_{leak} \]  

(18)

Where Q is the flow from the micro-pump. \( Q_{leak} \) is the internal leakage between the two chambers of the cylinder, \( v_c \) is the volume of the chamber of the output cylinder.

**IEHA Virtual Model**

Compactness of IEHA makes it difficult to measure the internal variables of the system, such as the micro pump eccentricity and internal pressure drop. Moreover, it is important to study the effect of configuring the internal variables (e.g. the dimensions of different mechanical parts) on the behavior of the system before modifying the system design in future prototypes. Therefore, development of a virtual model of the IEHA provides deeper understanding of the system and can be considered as an efficient platform for further system optimization. The effectiveness of the developed virtual model allows a quick and accurate evaluation of the internal parameters early in the design and development stage.

For this purpose, MATLAB-Simulink was used to model all the dynamic equations of the IEHA actuator, as well as fluid properties such as bulk modulus and internal leakage as shown in Appendix I. The blocks on the left-hand side of the figure represent the dynamics of the micro-valve, which takes signal X as an input, the resulting eccentricity E is applied to the micro-pump which is simulated as 15 individual micro-pistons (middle blocks). The pressure produced by the micro-pistons is sent to the passive distributor which directs it to side A or B of the linear actuator (right hand side blocks). The electric motor has been modeled as constant input rotating at a fixed speed. The used parameters values are given in Table 1.

The following simulation results show the variation of internal parameters against a step input of 0.04 [cm] of the micro valve position X. Generally, the input command X decides how much the micro-pump should be opened and as the pressure in the chamber increases, the carriage moves and E changes. Figure 5 shows the response of the carriage eccentricity, E, while it follows the micro-valve displacement X. As seen in Figure 5, the time response of E is around 0.007 [s] and it follows the input X with a negligible delay.

In order to illustrate the pressure variation in the micro-pistons, the variations of the output cylinder pressures are linked to the oscillation of flow. By looking at pump flow curve (Figure 6), it is clear that the oscillator with a frequency of 10 [Hz]. This oscillation is due to the fluid rippling from the micro-radial pump. There is also a slight peak in the beginning, the faster the eccentricity the bigger this peak of flow is.

In the beginning of the cylinder’s movement as shown in Figure 7, the pressure across the cylinder increases due to fluid compressibility. And as the load starts to move, the pressure difference across the cylinder drops and the velocity reaches the stable value of 2.2 [cm/s] as shown Figure 8. The pressure difference across the output stays positive (0.8 bar). This is
Table 1: The parameter values used in the virtual model of IEHA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_s$</td>
<td>Supply pressure of micro-valve</td>
<td>10 bar</td>
</tr>
<tr>
<td>$r_{mc}$</td>
<td>Micro-valve radius</td>
<td>0.25 cm</td>
</tr>
<tr>
<td>$m_c$</td>
<td>Carriage mass</td>
<td>0.091 kg</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Carriage active surface area</td>
<td>1.644 cm²</td>
</tr>
<tr>
<td>$E_{max}$</td>
<td>Maximum eccentricity</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Chamber volume of carriage</td>
<td>0.0822 cm³</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Bulk modulus</td>
<td>800 MPa</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Vena contracta coefficient</td>
<td>0.62</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
<td>840 kg/m³</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Electric motor rotational speed</td>
<td>3000 rpm</td>
</tr>
<tr>
<td>$P_{rp}$</td>
<td>Supply pressure of micro-pump</td>
<td>10 bar</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of micro-pistons</td>
<td>15</td>
</tr>
<tr>
<td>$r_{rp}$</td>
<td>Micro-pump in-out radius</td>
<td>0.2 cm</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Micro-piston mass</td>
<td>0.45 g</td>
</tr>
<tr>
<td>$S_p$</td>
<td>Micro-piston surface area</td>
<td>0.197 cm²</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Micro-piston stroke</td>
<td>0.27 cm</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Interior ring radius</td>
<td>0.3 cm</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between chamber</td>
<td>0.4 cm</td>
</tr>
<tr>
<td>$m$</td>
<td>Load mass</td>
<td>10 kg</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Chamber volume of output</td>
<td>2.356 cm³</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Output cylinder surface area</td>
<td>2.356 cm²</td>
</tr>
</tbody>
</table>

Figure 6: Flow produced by the micro-radial pump.

Figure 7: Pressure difference in the chambers of the output cylinder, $P_{cylA} - P_{cylB}$.

because of the asymmetry of the output piston ScylA which is almost half ScylB. This will lead also to a gradual change in the output piston position $Y$ as shown in Figure 9. This is continued till the occurrence of a change in input of the micro-valve signal.

In these results the output flow of the pump seems to immediately increase the pressure force in the output cylinder and make the piston velocity increase to a stable value. This is because the dynamics of the passive distributor are not taken into account in this model in Simulink.

The input/output model

Dynamic Equations

The ultimate goal of this section is to estimate the model order of the IEHA. As will be seen in subsection 8, this order combined with the test bench, are used to find a linearized function of the proposed IEHA, through system identification methods. Since our goal is to find the simplest low order form of equations equivalent to the system, internal leakage has been neglected in the following analysis. This is due to the compactness of the system.

Returning to the equations in subsection 5-5.3, if $P_c$ is derived and replaced from Equation 17 into Equation 18, and by considering the macroscopic value of flow according to Equation 8, we will have the eccentricity $E$ expression in terms of $F_{ext}$, $Y$, and their derivatives:

$$ E = \frac{S}{2NS_c \omega} \ddot{y} + \frac{1}{2NS_c \omega} v \frac{1}{2S_c} mY^{(3)} + \frac{1}{2NS_c \omega} v \frac{1}{2S_c} F_{ext} $$  (19)

Where $Y_3$ and $Y_5$ are the payload jerk and the second derivative of the jerk respectively.

On the other hand, the force of the micro-pump pistons exerted on the carriage chambers is given by Equation 16. By replacing $P_c$, the micro pump’s force can also be written in terms of the output position, the external force, and their derivatives:

$$ F_p = \left( P_{rp} - \frac{1}{2S_c} \left( m\ddot{y} + F_{ext} \right) \right) S_c \cos(\omega t) $$  (20)
The pressure difference in carriage chambers can be written in terms of \( \zeta \) and \( F_p \) according to Equation 12:

\[
\zeta = \frac{1}{2S_c} \left( m_\theta \dot{E} - F_p \right) \quad (21)
\]

Deriving \( \dot{E} \) from Equation 19, \( \zeta \) can also be written in terms of \( Y \) and \( F_{\text{ext}} \):

\[
\zeta = \frac{1}{2S_c} \left( mS_e Y'' + \frac{mS_e}{2N_S_\omega} \beta 2S_c Y'' + \frac{mS_e}{2N_S_\omega} \beta 2S_c F'' \right) + \left( P_p - \frac{1}{2S_c} \left( m\ddot{Y} + F_{\text{ext}} \right) \right) \frac{S_e \cos(\omega t)}{N_\omega} \quad (22)
\]

\( F_{\text{ext}}(3) \) is the third-time derivative of the external force while \( Y(3) \) and \( Y(5) \) are the payload jerk and the second derivative of the jerk. By replacing the Bernoulli equation of flow of the micro-valve (Equation 9) in the dynamic equation (Equation 12), we can write an expression which relates the input variable \( X \), to the internal variables \( E \) and \( \zeta \):

\[
2\pi C_{r_0} \sqrt{\frac{\rho}{P}} \left( X - E \right) \frac{P_p}{2} \frac{\dot{E}}{P} = S_c \dot{E} + \frac{\dot{\theta}}{\beta} \zeta \quad (23)
\]

Replacing \( E \) and \( \zeta \) from Equation 19 and Equation 22 in the latter, we will obtain a non-linear equation that represents the relationship between the system input, micro-valve position \( X \), and the measurable values at the end-effector, \( Y \), \( F_{\text{ext}} \), and their derivatives.

\[
X = A + \frac{B}{C} \quad (24)
\]

Where,

\[
A = K_{10} \ddot{Y} + K_{11}m Y'' + K_{11}F_{\text{ext}}
\]

\[
B = K_{3} \left( K_{10} \ddot{Y} + K_{11}m Y'' + K_{11}F_{\text{ext}} \right) + K_{9} \left( K_{13} Y^{(4)} + K_{14} m Y'' + K_{14} F_{\text{ext}} \right) - \left( K_{16} + K_{17}\dot{Y} + K_{17} F_{\text{ext}} \right) \sin(\omega t) + K_{18}\cos(\omega t) Y(5) + K_{14}\cos(\omega t) F_{\text{ext}}
\]

\[
C = K_{e} \sqrt{1 + K_{1} Y^{(3)} + K_{2} \cos(\omega t) + K_{3} \sin(\omega t) + K_{4} F_{\text{ext}}}
\]

The coefficients \( K_1 \) to \( K_{20} \) are all constants and depend on the dimensions of the actuator and the characteristics of the fluid. These coefficients are given as follows:

\[
K_1 = \frac{mS_e}{4PS_{SP}N_\omega} \quad K_{1m} = \frac{m \dot{\theta} m}{4\beta S_{SP}N_\omega}
\]

\[
K_2 = \frac{mS_e}{8PS_{SP}N_\omega} \quad K_3 = \frac{S_p}{2S_c} \quad K_{4m} = -\frac{S_p m}{4S_{SP}P} \quad K_4 = -\frac{S_p}{4S_{SP}P}
\]

\[
K_7 = 2\pi C_{r_0} \sqrt{\frac{\rho}{P}} \frac{\dot{\theta}}{\beta} \quad K_8 = S_c \beta \quad K_9 = \frac{\dot{\theta}}{2S_c}
\]

\[
K_{10} = \frac{S_c}{2S_pN_\omega} \quad K_{11m} = \frac{m \dot{\theta} m}{4\beta S_{SP}N_\omega}
\]

\[
K_{11} = \frac{\dot{\theta}}{4\beta S_{SP}N_\omega} \quad K_{13} = \frac{S_{SP}m}{2S_pN_\omega} \quad K_{14} = \frac{v_m}{4\beta S_{SP}N_\omega}
\]

\[
K_{15} = \frac{\dot{\theta}}{4\beta S_{SP}N_\omega} \quad K_{16} = -Ps_{SP} \omega \quad K_{17m} = \frac{S_{SP}m}{2S_c}
\]

\[
K_{17} = \frac{S_{SP} \omega}{2S_c} \quad K_{18m} = \frac{S_{SP}m}{2S_c} \quad K_{18} = \frac{S_p}{2S_c}
\]

**Model Reduction**

By combining the coefficients in Equation (24), the system is found to be non-linear and of order six. However, with an analytical study of the coefficients of the high order derivatives. Some of these terms are of very low orders of amplitude compared to the others. These terms can be neglected without having a major effect on the behavior of the actuator performance.
Identification of the Linear System

MATLAB system identification toolbox has been used to estimate a linear transfer function that best describes the behavior of the system. This toolbox estimates model parameters using iterative prediction-error minimization method (PEM). PEM is an iterative estimation command, using non-linear least squares algorithm to minimize the cost function.

The system input-output linear function has been identified with variable load (can also be interpreted as being in contact with the environment). Unlike simple micro-valves, dynamics of IEHA depends on the load (this is also the property that makes it back-drivable). Increase in the external force, causes increase of the cylinder pressure, which will increase the pressure in the micro-pistons. Consequently, the oil pressure on the carriage goes up and this can displace the carriage, (i.e. modify the eccentricity E). Hence, the output flow of the micro-pump is changed. This effect is also seen in the analytical relation between Y and X. Therefore, we consider Fext as an input to our system. Different external forces were applied to the output stroke. These forces included a step and a chirp signal of maximum frequency of 10 Hz, with amplitude of 100 ~ 1000 N. Inserting X and Fext as two separate inputs to the system, a new linear model has been estimated as follows:

\[ Y(s) = G_1(s)X + G_2(s)F_{\text{ext}} \]  

\[ G_1(s) = \frac{K}{s(1 + \lambda_1s)(1 + \lambda_2s)(1 + \lambda_3s)} \]  

\[ G_2(s) = \frac{K'}{s(1 + \lambda_2s)} \]

Where

\[ \lambda_1 = 10^{-3}[1.14, 1.14, 1.11] \]

\[ \lambda_2 = 0.43 \times 10^{-6} \]

\[ K = 115.4 \]

\[ K' = 6.3 \times 10^{-4} \]

The system found here is of fourth order, which corresponds to our findings in the previous section. To illustrate the accuracy of the model above, the inputs and the output of the virtual model and the linear model, for chirp signal inputs are shown in Figure 11. The output response of the linear system Y is found to be the same as of the virtual model.

To validate the piston response, another simulation is conducted. A step input of 0.04 [cm] is given to the micro-valve, then after 0.5 [s], the micro-valve is closed and the output is suspended at a certain position. Then a varying sinusoidal force is applied on the cylinder, which will pull and push the piston. The results are shown in Figure 13.
From those two previous simulation results, the virtual model was compared with the linear model while responding to step, chimp and sinusoidal inputs and proved its worthiness. This validates the response of the reduced order model when compared with the full order one.

**Predictive controller implementation**

Many control schemes have been proposed for the control of hydraulic actuators. Simple PID controllers have proven to be inadequate, because of complex non-linearities of hydraulics. Specifically, in the proposed IEHA, the controller signal is very sensitive to the dynamics of the internal parameters, and this makes the PID tuning complicated. Hence, another control method is sought after to control the position of the load actuator.

Since, we have already a linear based model for the IEHA in addition to the virtual model; a model predictive control algorithm

![Figure 11: Comparison of the outputs from the linear model of equation (26) and the virtual model, for step input.](image1)

![Figure 12: Predictive control block diagram.](image2)

![Figure 13: (a) sinusoid X input, (b) external force $F_{ext}$ input, (c) Simulated position tracking of loaded actuator.](image3)
seems adequate. The model predictive control is a strategy that is based on the explicit use of some kind of linear system model to predict the controlled variables over a certain time period [30]. In our case the linear model of the IEHA can be the predicting model. This way we avoid the undesirable dynamics of the actuator in the feedback. Another advantage of this method is that this feedback is faster than the one coming from the nonlinear model. Therefore, the control signal is adjusted by “predicting” the future error. The control block diagram is shown in Figure 12.

**Simulation Results**

In the simulation results shown in Figure 13, the position trajectory is a sine wave with frequency $\pi \, \text{rad/s}$, a sinusoidal force of 600 [N] is applied on the piston. As we can see in Figure 11c, the control signal is modified to compensate for this change in the load and keep the actuator in the commanded position. The resulting response has almost no delay time, negligible overshoot, and insignificant steady state error.

**Conclusion**

In this paper a detailed mathematical model and a virtual model for the IEHA have been developed. The system order is obtained and simplified by neglecting the ineffective parameters. An analytical study of IEHA combined with black-box identification has been used to find the linear model of this highly complex system. This linear model is used in implementing a predictive controller to show its precision in position tracking. Since the IEHA main application is related human-robot interaction, the external force exerted by the environment is taken as an input of the system.

For future work, the internal parameters of the hydraulic mathematical model such as leakage and compressibility are not constant and vary with time and temperature. Hence, a further work must consider a better solution to identify the internal states of the system. A better use of an adaptive controller accordingly can be pursued. On the other hand, the passive distributor model has to be included in order to get a complete description of the IEHA model. The new virtual model presented can be used for manufacturing other actuator prototypes with all of its inner parameters identified. Finally, the hardware use of the IEHA to actuate an active compliant hydraulic arm is under development. This will enable us to have an integrated hydraulic actuator for the underdevelopment humanoid robot HYDROID.

**Acknowledgement**

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**References**


Appendix I: Schematic View of the Virtual Model in Matlab-Simulink.